

CS1800

Fall 2025

Recitation 1 - Intro to Discrete Math

September 10 & 11, 2025

Recitations

Normally, CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

However, no homework is assigned yet! For today, we'll introduce a couple of problems/puzzles to get a feel for the type of work we do here in CS1800/CS1802.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to get to know them, and to learn more about discrete math.

Intro/Puzzle 1

You can work in teams of any size for this problem. We live in a universe where we have exactly four symbols:

1
2
3
.

We want to create numeric values with these symbols, following some rules: we can use up to four symbols, we can put the symbols in any order, but we can't reuse any symbol in a given number. Numbers still work in the usual way, e.g., we can create the values 1.3 or 12.3 within these rules.

With your team, answer these questions?

1. What's the smallest number we can make?
2. What's the largest number we can make?
3. How many different numbers can we make?

Solution

1. Smallest -- $.1$
2. Largest -- 321
3. Total -- 48.

Order them based on what symbol they start with:

- Decimal: $.1, .12, .123, .13, .132, .2, .21, .213, .23, .231, .3, .31, .312, .32, .321$ (15)
- Starts with 1: $1, 1.2, 1.23, 1.3, 1.32, 12, 12.3, 13, 13.2, 123, 132$ (11)
- Starts with 2: $2, 2.1, 2.13, 2.3, 2.31, 21, 21.3, 23, 23.1, 213, 231$ (11)
- Starts with 3: $3, 3.1, 3.12, 3.2, 3.21, 31, 31.2, 32, 32.1, 312, 321$ (11)

Total: $33+15 = 48$

Intro/Puzzle 2

This game is for teams of two. With your partner, write down these numbers: **1, 2, 3, 4, 5, 6, 7, 8, 9**

Take turns with your partner; you each select one number at a time and then cross it off the list. Your goal is to be the first to have **three numbers that sum to 15**.

Play this game with your partner a few times, alternating who starts. Record the number of times each person wins, and the number of times you end in a tie (i.e., nobody gets to fifteen).

You may have played this exact game before, maybe as a kid, but with a different format and a different name. It's [this game](#)!

Questions for your team:

1. Take those same numbers (1, 2, 3, 4, 5, 6, 7, 8, 9) and arrange them in a 3x3 square such that each row, column, and diagonal sums to 15.
2. Explain how this game is related to (in fact, basically the same thing as) the childhood game linked above.

Solutions:

1. There are a few ways to do this! Here are two:

2	7	6
9	5	1
4	3	8

2	9	4
7	5	3
6	1	8

2. We win at tic-tac-toe by having the same symbol (X or O) across a row, a diagonal, or a column.

There are eight ways to win at tic-tac-toe:

- Win a row (3 ways to do this)
- Win a column (3 ways to do this)
- Win a diagonal (2 ways to do this)

There are also eight ways to win the game where we sum-to-15. You can select any of these combinations:

- a. 6, 7, 2
- b. 9, 5, 1
- c. 4, 3, 8
- d. 2, 9, 4
- e. 7, 5, 3
- f. 6, 1, 8
- g. 4, 5, 6
- h. 2, 5, 8

And those eight options appear in ANY solution to Part 1, and they are the same ways we win at tic-tac-toe.

So let's pick this board with the 15 numbers:

2	7	6
9	5	1
4	3	8

Now we transform this into tic-tac-toe. Every time you select a number from 1..9, mark "X" or "O" off the corresponding square on the board. If you win the number game by getting three numbers that sum to 15, you'll also win at tic-tac-toe!

Intro/Puzzle 3

This game is for teams of 4. Once you've formed your team, choose among yourselves: you are labelled A, B, C, and D.

A, B, C, and D are all waiting for the Green Line at the Northeastern station, but when it finally arrives it's so crowded the conductor can't let everybody on. But you all are a team, so you either all get on or you all wait.

So the conductor says, ok, we're going to make this deal. I'm going to put hats on your heads, but you won't be able to see your own hat. If any one of the 4 of you can tell me what color hat you're wearing, all four of you can get on the train. Otherwise, you all are stuck at the station until the next green line comes along.

Your team must follow two rules:

- Rule #1: no talking, except stating the color of your hat
- Rule #2: You can state only the color of your OWN hat

There are 4 hats: 2 black and 2 white.

- You don't know in advance who will get which hat
- You need a plan that will be successful no matter how the hats are distributed.

Every group of four will have 2 black hats and 2 white hats. You can't see your own hat. Here are the rest of the constraints:

- A can't see anyone
- B can't see anyone
- C can see B's hat
- D can see C's hat and B's hat

No talking allowed, other than stating the final answer. The final answer has to be about your OWN hat.

Take a few minutes and see if you can come up with a solution, no matter how the hats are distributed

Solution

- D can see B and C. If they are both wearing the same color, then D can state that my hat is the other color.
- C can see B. If D doesn't say anything after a minute, then C must know that B has a different color hat, so she can state the color of her own hat.

This is the type of problem we want to be able to solve in this class. What makes it 'discrete'?

- For a particular person, their hat is either black/white. No ambiguity.
- We need a solution that doesn't know the inputs in advance.

- We need our solution to work all the time.
- We have a strategy of the form: IF B and C are wearing the same color, THEN D's hat is the other color
 - Discrete math and proofs that we'll do in this class often look like that. If, then.
 - If a number is prime, then it is odd.
 - If a divides b, then b divides a