CS1800
9/9 - Tue.

Admin
- rec starts!
- pls say name
- lec question form

Agenda
1. Propositional Logic + operators
2. Implications
3. English → Logic, negating

Squishy ⇝ Formal

respect the original statement

1. Propositional Logic + operators

↳ propositions
logic statements
- meaningful
- declarative
- has truth value (T/F)

(x)

P	Prof. Patt teaches CS2000	T
Q	15 + 8 = 13	F
R	Grizz weighs ≥ 60 pands	T
S	Eat your food, Grizz!	n/a
T	x + 0 = x	n/a
U	For all real numbers x + 0 = x	T
	If x is a real number, then x + 0 = x	T

combine simple statements **operators** ¬ ∧ ∨

not ¬ ~

P has truth value
¬P has other truth value

English: not P
it is not the case that ...
negation of P

(ex) Grizz weighs in at 65 pands

R	¬R
T	F

Grizz weighs in at 59 pands

R	¬R
F	T

and ∧ P, Q logic statements
P ∧ Q logic statement

English: P and Q, P but Q

(ex) P = I am loud, Q = I am wrong
I'm loud but wrong!

truth table: all possible inputs
list outputs

(or) $\vee$ P, Q logic statement>
P ∨ Q logic statement

English: or, and/or

(ex) P the number x is prime
Q the number x is odd
P ∨ Q x is prime or odd
want True: 2, 3, 5, 9, 13, 15, ...

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

and: True when P, Q both True
otherwise, false!

or: false when P, Q both false
otherwise, true!

2. Implications

equivalent → formal
respect the original statement

basic operators: and / or / not
$\wedge$ $\vee$ $\neg$

(ex) P = it's my birthday
Q = we have cake!

want to convey:
- on 3/27, we have cake!
- not my bday, maybe cake maybe not
- bday, no cake?? no way

P	Q	?
T	T	T
T	F	F
F	T	T
F	F	T

$P \Rightarrow Q$

implication
English: If P, then Q
Q if P
whenever P, Q

(ex) If it's night,
P
then it's dark outside
Q

$P \Rightarrow Q$

P	Q	P ⇒ Q	
T	T	T	– sure
T	F	F	– lies!
F	T	T	– could be eclipse?
F	F	T	– sure

(ex) P = Lanzy wins the lotto
Q = Lanzy gets $1.8 billion

P ⇒ Q If P, then Q
Q if P
Q whenever P

P	Q	P ⇒ Q	
T	T	T	– great!
T	F	F	– got stiffed!
F	T	T	– stole it?
F	F	T	– normal life

3. English → Logic, negation

$\wedge$, $\vee$, $\neg$, $\Rightarrow$

and or not implies

① implication without $\Rightarrow$

② implications to each other?

③ $\neg(A \vee B)$ $\neg(A \wedge B)$? ✗

→ $V \Rightarrow R$

V	R	V⇒R
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	P⇒Q	¬P	¬P∨Q
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

by definition

$P \Rightarrow Q$ same as $\neg P \vee Q$

If it wasn't the lead pipe, then it wasn't Prof. Plum

If it wasn't Prof. Plum, then it couldn't have been the lead pipe

V	R	¬V	¬R	¬V⇒¬R	¬R⇒¬V	V⇒R
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Takeaways:

- truth table: one column per operator is a proof!
- $\neg R \Rightarrow \neg V$ has same outputs as $V \Rightarrow R$
  - ↳ same outputs for every input, logically equivalent