CS1800
10/7 -Tues.

Admin

- Quiz 1 graded :)
- HW3 out, due 10/14 9pm

Agenda

1. Set Functions Cont'd
2. Counting we've seen
3. Product rule, Sum rule
4. Breaking down problems

How many ways are there...?

0. Set Review

$A = \{1,2,3\}$ $B = \{3,4\}$ $C = \{2,3,5\}$

$|A \cup B|$? 4 $\{1,2,3,4\}$ PIE ✓

$|A \cup B \cup C|$? 5 $\{1,2,3,4,5\}$ PIE ✓ formula

$|A| + |B| - |A \cap B|$

$\emptyset$

$P(B)$? $\{\{\}, \{3,4\}, \{3\}, \{4\}\}$

- all possible subsets of B
- $\{\}$ subset of everything
- every set is a subset of itself

1. Set Functions Cont'd

↳ powerset, cartesian product, DIY

powerset

- input: set
- output: collection

$P(S) = \{A \mid A \subseteq S\}$

Cartesian product $\times$

- input: two sets
- output: set of ordered pairs

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

↳ order matters! $(a,b) \neq (b,a)$

ex evening at strangehouse

$L = \{\text{weds}, \text{rookie}, \text{matlock}\}$

$T = \{\text{archer}, \text{family guy}\}$

$L \cap T = \{\}$ disjoint

order matters: Lacey show, then a Tam show

$L \times T = \{(w,a), (w,fg), (rook,a), (rook,fg), (mat,a), (mat,fg)\}$

- Lacey show first, then Tam show $(w,a) \neq (a,w)$
- How many ways to spend an evening? 6 ways
  - task 1: pick Lacey show (3 ways)
  - task 2: pick Tam show (2 ways)

total: $3 \cdot 2 = 6$

2. Counting we've seen

↳ Cartesian product $|A \times B| = ?$ when $|A| = 3, |B| = 2, |A \times B| = 6$

what if A, B are not disjoint?

↳ $A = \{1, 2, 3\}$
$B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$
A B

$|A| = 3, |B| = 2 \quad |A \times B| = 3 \cdot 2 = 6$

↳ $A = \{a, b\}$
$B = \{a, b\}$

$A \times B = \{(a,a), (a,b), (b,a), (b,b)\}$

$|A| = 2 \quad |B| = 2 \quad |A \times B| = 2 \cdot 2 = 4$

in general... for any A, B $|A \times B| = |A| \cdot |B|$

- task 1: pick an element from A
- task 2: pick an element from

Cardinality of powerset $B = \{3, 4\}$ $P(B) = \{\{\}, \{3\}, \{4\}, \{3,4\}\}$

$|P(B)| = 4$

↳ $P(C) =$ what about $C = \{3, 4, 5\}$

$|P(C)| = 8$

$\{\{\}, \{3\}, \{4\}, \{3,4\}, \{5\}, \{3,5\}, \{4,5\}, \{3,4,5\}\}$

ex) $B = \{3, 4\}$ 1 = yes 0 = no

3	4	subset
1	1	$\{3,4\}$
1	0	$\{3\}$
0	1	$\{4\}$
0	0	$\{\}$

counting tasks in powerset
- for every element in set
- task: decide if it belongs in subset
- 2 ways for each task (1/0)

in general... $|P(S)| = 2^{|S|}$

3. Product Rule, Sum Rule

$|A \times B|$, $|P(S)|$ are examples of a general formula!

Product Rule (fundamental principle of counting)
- one task in n ways
- another task in m ways

(do) 2nd (multiply!)

- there are $n \cdot m$ ways to do task 1 and task 2

Sum Rule

- one task in n ways
- another task in m ways
- there $n + m$ ways to do task 1 or task 2

What other phrases to look for?

next - slides

4. Breaking down problems

↳ passwords with alphanumeric (36)
repetition is ok

How many of length 5 have at least one number?

↳ cases
- exactly one #
- exactly two #s
- :
- exactly five #s

↳ cases are annoying! Too many!
Instead, subtract anything invalid

total # - invalid cases

$36^5 - 26^5$

$= 48,584,800$

total # pwds of length 5: 36^5

How many of length 5...

- have only letters? 26^5
- start with a number?

$10 \cdot 36^4$

Counting problems...

at least / at most — cases
maybe subtraction?

and — multiply

or — add

different
and then
same or
in any order
— repetition ok?
order matters?