CS1800
11/7 - Fri :)

Admin
- HW6 due 11/13 9pm
- 11/11 no class, no OH
- Quiz 3 Fri 11/14

Agenda
1. Induction overview
2. Structure of a proof
3. Proving divisibility

Hint HW6 P5:
Sequence a_i = # games up to day i
consider: range of possible values
separate sequence for consecutive games
↳ range of possible values

1. Induction Overview → Proof by mathematical induction

Types of proofs in CS1800:
- truth table
- laws of logical equivalence
- laws of set equality
- proof by counter example
- proof by contrapositive

* mathematical induction
↳ used IRL!

Categories
1. value of a summation *
2. divisibility / number theory
3. Inequalities / growth of functions
4. Structural (sets, sequence, graphs)
5. Program correctness

last time $1, 2, 3, 4, \ldots, n$ seq
add values together, what is formula?

$$\begin{array}{cccc} 1 + & 2 + 3 + 4 + & \ldots & + n \\ n + & n-1 + n-2 + & \ldots & + 1 \\ \hline (n+1) & (n+1) & & (n+1) \end{array} \qquad \frac{(n)(n+1)}{2}$$

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

If we had all day, all the interest...

	$1+2+3+\ldots+n$	$\frac{(n)(n+1)}{2}$	
if $n=1$	1	$\frac{(1)(2)}{2} = 1$	✓
$n=2$	$1+2=3$	$\frac{(2)(3)}{2} = 3$	✓
$n=3$	$1+2+3=6$	$\frac{(3)(4)}{2} = 6$	✓

. . .

Induction is shorthand for that ↑

Review the $\frac{(n)(n+1)}{2}$ proof:

- predicate is a generalization no truth value!
- predicate into logic statement:
  1. plugin values $P(1), P(2)\ldots$
  2. use quantifiers $\forall, \exists$

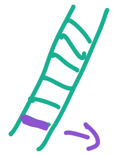
live where $P(k)$ is true

when we apply it, the IH is used!

1. Is it convincing? ☑
2. why do we have predicate, logic statement?
3. What is the purpose of the implication?
4. When/why do we say "by inductive hypothesis"?

$P(k) \Rightarrow P(k+1)$

- create a universe where $P(k)$ is true
- show $P(k+1)$



if on step k, you can get to step k+1

first step: base case $P(1)$

2. Structure of a proof

1. Predicate 2. Logic Statement 3. Base case 4. Inductive step $P(k) \Rightarrow P(k+1)$

Predicate $P(x)$ $\sum_{i=1}^{x} i = \frac{(x)(x+1)}{2}$

Logic Statement $\forall n \in \mathbb{Z}^+ \; P(n)$ for which values is predicate true?

Base Case $P(1)$ $\sum_{i=1}^{1} i = 1$ $\frac{(1)(1+1)}{2} = 1$

definition of summation == formula!
first rung of ladder
plug in lowest value we care about

Inductive Step $P(k) \Rightarrow P(k+1)$

Assume $P(k)$, $\sum_{i=1}^{k} i = \frac{(k)(k+1)}{2}$ IH

we live where $P(k)$ is true!
↳ for some arbitrary $k \geq 1$, the formula works IH

Goal: use IH to show $P(k+1)$

$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

goal: show $P(k+1)$ using $P(k)$

$\sum_{i=1}^{k+1} i = 1+2+3+\ldots+k+(k+1)$

write out $P(k+1)$ by definition

$$\sum_{i=1}^{k+1} i = 1+2+3+\ldots+k+(k+1)$$

$$= \sum_{i=1}^{k} i + (k+1)$$

def of summation

$$= \frac{(k)(k+1)}{2} + (k+1)$$

by IH

b/c in our universe $P(k)$ is true! $\sum_{i=1}^{k} i = \frac{(k)(k+1)}{2}$

$= \frac{(k)(k+1)}{2} + \frac{2(k+1)}{2}$

$= \frac{k^2+k+2k+2}{2}$

$= \frac{k^2+3k+2}{2}$

$= \frac{(k+1)(k+2)}{2}$ done!

goal: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

In HW/quiz/final:
- 4 steps
- P(k) the IH
- P(k+1) the goal
- when apply the IH

3. Induction - Divisibility

Predicate P(x) — 6^x+4 is a multiple of 5

$5 \mid 6^x+4$ "5 divides 6^x+4"

Logic statement — $\forall n \in \mathbb{Z}^+ \; P(n)$ * start with $\mathbb{Z}^+$, maybe adjust?

Base case — Plug in $x=1$ P(x) * plug in x, generate a value 6^x+4, see if it's 5 · something

P(1) gives us $6^1+4=10$

↳ $10 = \underline{5} \cdot 2$

Inductive Step — $P(k) \Rightarrow P(k+1)$

IH P(k) 6^k+4 is a multiple of 5

$5 \cdot \text{something} = 6^k+4$

Goal P(k+1) show $6^{k+1}+4$ is a multiple of 5

Proof:

$6^{k+1}+4 = 6^k \cdot 6^1 + 4$ # exponent w/same base

$= 6^k \cdot 6 + 4$ # 6^k, 4 useful b/c part of IH

$= 6^k(5+1) + 4$ # $6 = 5+1$

$= 5 \cdot 6^k + 6^k + 4$ # multiply through

$= 5 \cdot 6^k + 5 * z$ # for some int z; by IH ☆☆ :)

$= 5(6^k + z)$ # factor out the 5

↳ 5 * something, so $6^{k+1}+4$ is a multiple of 5 :)