CS1800
11/14-Tues.

Admin

- HW5 due 9pm
- HW6 out, due 11/13 9pm
- Quiz 3 11/14 in class
- no class 11/11

Agenda

1. Summations
2. Summation Formulas
3. Summation math

0. Review

- sequences vs. sets
  - ordered $\{a_n\}$ $1,2,3 \neq 3,2,1$
  - repeats ok $\{a_n\}$ $-1, 1, -1, 1, -1, \ldots$
- arithmetic sequence $a_k = a + d(k-1)$
- geometric sequence $a_k = a \cdot r^{k-1}$

1. Summations

$\sum$ sigma

- add together terms
- starting value
- stopping value

by def...

$$\sum_{i=1}^{5} i = \begin{matrix} i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=5 \end{matrix} = 1+2+3+4+5$$

$$\sum_{i=1}^{5} 3i = 3\cdot1+3\cdot2+3\cdot3+3\cdot4+3\cdot5$$
$$= 3+6+9+12+15$$

$$\sum_{i=1}^{n} i = 1+2+3+4+\ldots+(n-1)+n$$

goal: $\boxed{\text{what's the answer?}}$ slow way !!

$$\sum_{i=1}^{5} i = 1+2+3+4+5 = 15$$

$$\sum_{i=1}^{5} 3i = 3+6+9+12+15 = 45$$

- given n, formula
- given n = value, single number

ex first 100 positive ints

$$\sum_{i=1}^{100} i = 1+2+3+\ldots+99+100$$

shortcut by Gauss

	1	2	3	... 99	100
+	100	99	98	2	1
	101	101	101 ...	101	101

first	last	
1	100	100+1=101
2	99	99+2=101
3	98	98+3=101
...		

↳ value 101 (first + last)
100 times
doubled the sequence

$$= \frac{(101)(100)}{2} = 5050$$

general formula for first n positive integers $\sum_{i=1}^{n} i$

value $n+1$
n times
doubled the sequence

$$\begin{array}{cccccc} 1 & 2 & 3 & \ldots & (n-1) & n \\ + \; n & (n-1) & (n-2) & \ldots & 2 & 1 \\ \hline n+1 & n+1 & n+1 & & n+1 & n+1 \end{array}$$

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

Applying the formula vs. labor-intensive

$$\sum_{i=1}^{6} i = \frac{6 \cdot 7}{2} = 21$$

$1+2+3+4+5+6 = 21$ ☑

$$\sum_{i=1}^{5} i = \frac{5 \cdot 6}{2} = 15$$

$1+2+3+4+5 = 15$ ☑

Patterns in other sequences? ex positive, odd integers

$1+3+5+\ldots+99$

#1 in sigma notation?

$$\sum_{i=1}^{50} (2i-1)$$

$i=1 \quad 2 \cdot 1 - 1 = 1$

$i=2 \quad 2 \cdot 2 - 1 = 3$

$i=3 \quad 2 \cdot 3 - 1 = 5$

$i=4 \quad 2 \cdot 4 - 1 = 7$

...

$i=50 \quad 2 \cdot 50 - 1 = 99$

#2 what is pattern?

$a_1 = \sum_{i=1}^{1} (2i-1) = 1$

$a_1 + a_2 = 1+3 = 4$

$a_1 + a_2 + a_3 = 1+3+5 = 9$

$a_1 + a_2 + a_3 + a_4 = 1+3+5+7 = 16$

$$\sum_{i=1}^{50} (2i-1) = 50^2 = 2500$$

sum of first n odd positive integers

$$\sum_{i=1}^{n} (2i-1) = n^2$$

2. Summation Formulas → more general than the 2 so far

- arithmetic sequence
- summation of terms

first n positive ints $\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$

↳ n = # terms

$n+1$ = first term + last term
(1) (100)

double the sequence

any arithmetic seq:

$$\frac{(n)(\text{first} + \text{last})}{2}$$

Formula vs. known values → first 50 odd positive ints $= 50^2 = 2500$

↳ $n = 50$
first = 1
last = 99

$\frac{(50)(99+1)}{2} = \frac{5000}{2} = 2500$ ☑

!!

any geometric seq:

$$\frac{a \cdot r^n - a}{r-1}$$

↳ n = # terms
a = initial value
r = common ratio

Formula vs. labor intensive: $1+2+4+8+16+32+64+128$?

$a = 1$
$n = 8$
$r = 2$

$\frac{1 \cdot 2^8 - 1}{2-1} = 2^8 - 1 = 255$

!!

↳ by hand $= 255$

Ex sequence 2, 4, 6, 8, 10, 12, . . .

1. sum of first 12 terms?

$n = \boxed{12}$

first $= \boxed{2}$

last $= a_k = a + (k-1)d$

$a_{12} = 2 + (12-1)2$

$= 2 + 22$

$\boxed{= 24}$

2. sum of first n terms, simplified?

↳ formula $\frac{(n)(\text{first} + \text{last})}{2}$

$n = n$

first $= 2$

last $= a_n = 2 + (n-1)2$

↳ $\frac{(n)(2 + 2 + (n-1)2)}{2}$

$$\text{Sum} = \frac{(n)(\text{first} + \text{last})}{2} = \frac{(12)(2+24)}{2}$$

$= 156$

sanity check!

$n = 12$

formula vs. formula!

$$= \frac{(n)(2+2+2n-2)}{2}$$

$$= \frac{(n)(2+2n)}{2} = \frac{2n+2n^2}{2} = n+n^2$$

$n+n^2 = 12+144 = 156$!!

First n pos ints

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

First n pos odd ints

$$\sum_{i=1}^{n} (2i-1) = n^2$$

First n pos even ints

$$\sum_{i=1}^{n} 2i = n+n^2$$

3. Summation math → looks fancy just addition!

ex - split summation

$$\sum_{i=1}^{50} = \sum_{i=1}^{25} + \sum_{i=26}^{50}$$

ex - multiply

$$\sum_{i=1}^{n} 3i = 3 \cdot 1 + 3 \cdot 2 + 3 \cdot 3 + \ldots + 3n$$

$$= 3(1+2+3+\ldots+n)$$

$$= 3 \cdot \sum_{i=1}^{n} i$$

ex - adding

$$\sum_{i=1}^{n} (i-1) = (1-1)+(2-1)+\ldots+(n-1)$$

$$= (-1)(n) + (1+2+3+\ldots+n)$$

$$= -n + 1 + 2 + 3 + \ldots + n$$

$$= 1+2+3+\ldots+n-1$$

$$= \sum_{i=1}^{n-1} i$$

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

$$\sum_{i=1}^{n-1} i = \frac{(n-1)(n)}{2}$$

ex - breaking

$$\sum_{i=1}^{n} (ai+bi) = \sum_{i=1}^{n} ai + \sum_{i=1}^{n} bi$$

A sequence! 1, 3, 6, 10, 15, 21, 28, . . .

- last time $a_k = \frac{k+k^2}{2}$
- today $a_k = \sum_{i=1}^{k} i$

$a_1 = 1$

$a_2 = 1 + 2 = 3$

$a_3 = 1 + 2 + 3 = 6$

$a_4 = 1 + 2 + 3 + 4 = 10$

. . .

a_k = sum of first k positive ints

$5 + 10 + 15 + \ldots + 50$

in sigma notation?

$\sum_{i=1}^{10} 5i$

(3) $3 + 6 + 9 + \ldots + 60$

how many terms? -> 20

$\sum_{i=1}^{20} 3i$

$60/3 = 20$