CS1800
9/30-Tues.

Admin

- HW2 due 9pm
- Quiz 1 Fri!
- HW1 graded today

Agenda

1. Set operations
2. Set Equality Proofs
3. Set Functions

0. Set Review ~ from Fri

$\mathcal{U}$ = deck of cards

$A = \{x \mid x \text{ is red}\}$ $|A| = 26$

$B = \{x \mid x \text{ is } \diamondsuit\}$ $|B| = 13$

$C = \{x \mid x < 5\}$ $|C| = 12$

$D = \{2x \mid x \text{ is even} \wedge x < 6\}$ $|D| = 8$

↳ 4, 8 of every suit

Set characteristics
1. unordered
2. no dupes

what is cardinality of...

① x is red but $x \geq 5$

$A \cap \overline{C} = A - C$

$|A - C| = |A| - |A \cap C|$

$26 - 6 = 20$

② $x < 5$ or x is $\diamondsuit$ PIE

$C \cup B$

$|C \cup B| = |C| + |B| - |C \cap B| = 22$

$12 + 13 - 3$

③ $x \in A \wedge x \notin (C \cap D)$

$A \cap \overline{(C \cap D)} = A - (C \cap D)$

$|A - (C \cap D)| = |A| - |A \cap C \cap D|$

$26 - 2 = 24$

- result of set operation is a set

$\underline{A} \cap \underline{B}$ ~ whole thing is a set!
set set

- $|A \cup B| = |A| + |B| - |A \cap B|$

what if $|A \cup B \cup C|$?

1. Set Operations

↳ slides joinpd.com

2. Set Equality

↳ two sets are equal if they have all the same elements

$\{1, 2, 4\} = \{4, 2, 1\} = \{4, 4, 2, 1, 2\}$ ~ specific sets (not as interesting!)

$A = B$ $x \in A \Rightarrow x \in B \wedge x \in B \Rightarrow x \in A$

more interesting: after set operations
- start: specific sets

↳ $\boxed{\text{is the equality true for arbitrary sets or just in this example?}}$

- if always true... prove with set equality laws
- otherwise... counterexample to prove false

(ex) $A = \{x \mid x\text{'s first initial is D}\}$

$B = \{x \mid x\text{'s last initial is S}\}$

$\mathcal{U} = \{\text{Leroy Strange, Daniel Patterson}\}$

what are these in roster notation?

$\overline{(A \cap B)}$ $\{LS, DP\}$

$\{DP\}$ $\overline{B}$

$A \cap B$ = no one

$\overline{A \cap B}$ = everyone

$(A \cup B) - (A \cap B)$ $\{LS, DP\}$

$\{DP\}$ $\overline{(A \cap B)} - B$

Is the relationship always equal, or just in this example?

↳ If yes, prove using laws

If no, counterexample ~ add/remove elements to give an example where sets are not the same

$\overline{(A \cap B)} = (A \cup B) - (A \cap B)$?

not the same! → counterexample

$\mathcal{U} = \{DP, LS\}$ ~ add DS

$\mathcal{U} = \{DP, LS, DS\}$

$\overline{(A \cap B)}$ = DP, LS

$(A \cup B) - (A \cap B)$ = DP, LS

~ :|

another counterexample

$\mathcal{U} = \{DP, LS, IH\}$

$\overline{(A \cap B)}$: DP, LS, IH

$(A \cup B) - (A \cap B)$ = DP, LS

~ :)

$\overline{(A \cap B)} - B = \overline{B}$?

the same! :)

$\overline{(A \cap B)} - B$

$(\overline{A} \cup \overline{B}) - B$ — de morgan

$(\overline{A} \cup \overline{B}) \cap \overline{B}$ — def. of difference

$\overline{B}$ — absorption

done!

3. Set Functions

↳ powerset, cartesian product

$\cup, \cap, -$ operations

powerset

- input: set
- output: collection

what's a collection?

- set of sets

$C = \{ \{a, b\}, \{c\}, \{d, e\} \}$

$\{a, b\} \in C$ $\{ \{a, b\}, \{c\} \} \subseteq C$

$P(S) = \{A \mid A \subseteq S\}$

↳ all possible subsets of S

including...

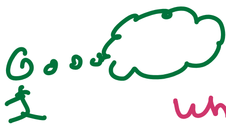
- empty set $\{\}$ (empty is subset of everything)
- S itself (every set is a subset of itself)

ex $S = \{5, 6\}$

$P(S) = \{ \{\}, \{5\}, \{6\}, \{5, 6\} \}$

ex $S = \{1, 2, 3\}$

Logic	truth value?
$\{1,2\} \in P(S)$	T
$\{1,2\} \subseteq P(S)$	F
$\{\{2\}\} \subseteq P(S)$	T
$\lvert P(S) \rvert = 10$	F



what is $\lvert P(S) \rvert$?