CS1800
9/26-Fri :)

Admin

- HW2 due 9/30 9pm
- Quiz 1 in class 10/3
- Sec 2 final exam
  Fri 12/12 7-10Am

Agenda

1. Set definitions
2. Set operations
3. Countability and PIE

0. Lecture Question

We were told we could have 1 page of notes for the quiz; will we be given the laws of logical equivalence or do we need to write them out ourselves?

- write 'em if you need 'em! :)

1. Set Definitions

↳ a set is a discrete structure
- hold multiple elements
- contain elements in $\{\}$

two main characteristics about sets
1. unordered
2. no duplicates

$A = \{2,4,6\} = \{6,4,2\} = \{6,6,4,2,4,6\}$

- roster notation: list each element individually

squishy → logic → sets

$\in$ is a member of
$\notin$ is not a member of
↳ logic statements!

$2 \in A$ T snickers $\in A$ F
$8 \in A$ F mars $\notin A$ T

universe: deck of cards ($\mathcal{U}$)
- 52 total cards
- 4 suits: clubs, diamonds, hearts, spades
- 13 values in each suit: 2, 3, 4, ..., 10, J, Q, K, Ace
  ↳ face cards
- diamonds, hearts: red
- spades, clubs: black

ex) $\{9H, 3C\} = \{3C, 9H\}$ (roster notation)

Set builder notation
↳ describe arbitrary element

$A = \{x \mid x \text{ is red}\}$ $C = \{x \mid x \text{ is Queen}\}$
$B = \{x \mid x \text{ is a face card}\}$

squishy
card that is not red, but is face

logic
$x \notin A$ not red
$x \in B$ face

set
$\{x \mid x \notin A \wedge x \in B\}$
(logic T/F) (logic T/F)

practice

1. card that is red or a face card
   set: $\{x \mid x \in A \vee x \in B\}$
2. red face card
   $\{x \mid x \in A \wedge x \in B\}$

ex)
in set: QS, 9D
not in set: 9S

in set: KH (♡ choice)
not in set: 3H, AD, 10S

3. red but not a face card $\{x \mid x \in A \wedge x \notin B\}$

in set: AH
not in set: 2S, QD, KC

4. Queen that's a face card $\{x \mid x \in B \wedge x \in C\}$

in set: QD QC, QH, QS

↳ Queen that's a face card but?
All Queens are facecards
If Queen, then facecard

$x \in C \Rightarrow x \in B$ → subset

implication in a set

Q	Fcard	Q ⇒ Face
T	T	T
T	F	F
F	T	T
F	F	T

For arbitrary sets S, T

subset $S \subseteq T$ def: $x \in S \Rightarrow x \in T$

Venn diagram

Queens ⊆ Facecards
Diamonds ⊆ Red
Red ⊆ Red * every set is a subset of itself
{} ⊆ Red * empty set is subset of everything!

proper subset $S \subset T$

$x \in S \Rightarrow x \in T \wedge S \neq T$

Queens ⊂ Facecards
Diamonds ⊂ Red
Red ⊄ Red Facecards ⊄ Facecards

2. Set Operations

↳ operators we apply to sets that generate a new set

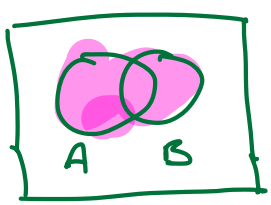
- definitions are logic based
- arbitrary sets S, T
- universal set $\mathcal{U}$

All we need: $\cup, \cap, -$

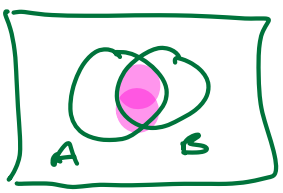
union $A \cup B$

- A is a set
- B is a set
- $A \cup B$ is a set

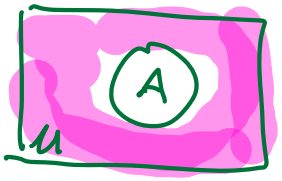
$A \cup B = \{x \mid x \in A \vee x \in B\}$



intersection $A \cap B$ $A \cap B = \{x \mid x \in A \wedge x \in B\}$

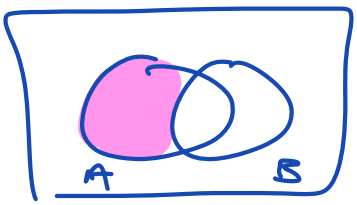


Complement $\bar{A}$ $\quad \bar{A} = \{x \mid x \in \mathcal{U} \wedge x \notin A\}$



Shortcut / convenience

difference $A-B$ $\quad A-B = A \cap \bar{B}$
$= \{x \mid x \in A \wedge x \notin B\}$



3. Cardinality PIE

↳ cardinality $|A|$ = # of distinct elements in A

↳ Principle of Inclusion / Exclusion PIE $\quad |A \cup B| = |A| + |B| - |A \cap B|$

ex $A = \{x \mid x \text{ is red}\}$ $B = \{x \mid x \text{ is face}\}$
$C = \{x \mid x \text{ is Queen}\}$

$|A| = 26$
$|B| = 12$ (J, Q, K of C, D, H, S)
$|C| = 4$

more interesting... cardinality after set operations

- How many red face cards? $|A \cap B| = 6$ (12 face, ½ are red)
- How many red, but not face? $|A - B| = |A| - |A \cap B| = 26 - 6 = 20$
- How many red or face? $|A \cup B| = |A| + |B| - |A \cap B| = 26 + 12 - 6 = 32$

↳ why the PIE?

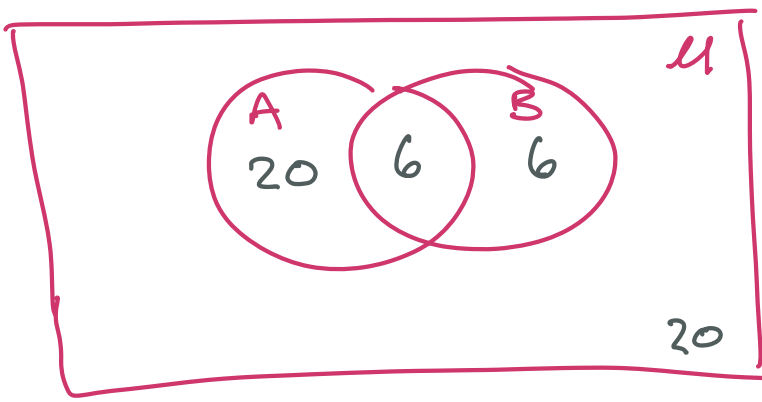
	3H	KS	2D	KD	KH
$\|A\|$ = red cards	✓		✓	✓	✓
$\|B\|$ = face cards		✓		✓	✓

- red face cards get double-counted !!
- subtract the intersection

$|A - B|$

$|A| - |A \cap B|$

in a Venn diagram, cardinalities are individual to the components



$|A| = 20 + 6 = 6$

goal: add cardinalities for all components and get $|\mathcal{U}|$

$|\mathcal{U}| = 20 + 6 + 6 + 20 = 52$

New sets!

$A = \{x \mid x \text{ is red}\}$ $B = \{x \mid x \text{ is diamond}\}$

$C = \{x \mid x < 5\}$ $D = \{2x \mid x \text{ is even} \wedge x < 6\}$

$|A| = 26$ (½ deck)

$|B| = 13$ (½ red)

$|C| = 12$ (2, 3, 4 of C, D, H, S)

$|D| = 8$

why?

$D = \{2x \mid x \text{ is even} \wedge x < 6\}$

$= \{4C, 4D, 4H, 4S, 8C, 8D, 8H, 8S\}$

Set operations and cardinalities

- red but ≥ 5 $A - C = A \cap \bar{C}$
- < 5 or diamond $B \cup C$
- $x \mid x \in A \wedge x \notin (C \cap D)$ $A - (C \cap D)$

- start with x is even $\wedge\ x < 6$
  2, 4 of C, H, D, S
- apply $2x$ to get
  4, 8 of C, H, D, S

$\{2x \in \mathcal{U} \mid x \text{ even} \wedge x < 6\}$