

CS1800
10/17 - Fri

Amin

- HW4 due 10/21 9pm
- Quiz 2 next Fri in class

Agenda

1. Pigeonhole Principle
2. Probability

lecture question

The quiz 2 content says sets and set operations as well as counting. What is the counting content out of? Will permutations combinations and stars and bars be included?

↳ good! HW3, HW4

sets, set operations, set equality

product, sum, combos, perms, stars + bars, overcounting

1. Pigeonhole Principle

counting \leftrightarrow probability

guarantees! (prob = likely, unlikely)

boxes \square
objects \circ

"have to" "must" "at least"

ceiling $\lceil \rceil$ round up to next integer

$$\lceil 2.7 \rceil = 3$$

$$\lceil 2.1 \rceil = 3$$

$$\lceil 2 \rceil = 2$$

$$\lceil 2.0001 \rceil = 3$$

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<slides>

2. Probability \rightarrow sets, counting

experiment: infinitely repeatable procedure with a well-defined set of outcomes


sample space S : all possible outcomes of an experiment
 $S = \{s_1, s_2, \dots, s_n\}$ $s_i = \text{one outcome}$

event space E : $E \subseteq S$ subset of outcomes we care about

$$E_1 = \{s_1\} \quad E_2 = \{s_2, s_3\} \quad E_3 = \{s_1, s_3, s_5\}$$

probability of event E $Pr(E) = \frac{|E|}{|S|}$

cardinality

ex) 6-sided die  experiment = rolling the die
 "fair" - all outcomes equally likely

$$S = \{1, 2, 3, 4, 5, 6\}$$

event examples

$$E = \{3\} \quad Pr(E) = \frac{|E|}{|S|} = \frac{1}{6}$$

$$E = \{1, 3, 5\} \quad Pr(\text{odd}) = \frac{|E|}{|S|} = \frac{3}{6} = \frac{1}{2} = 0.5$$

↳ roll a 1, 3, or 5 **Sum!**

$$Pr(1) + Pr(3) + Pr(5) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

ex) roll the die twice

Pr(4 then 5)? **product rule!**

$$Pr(4) = \frac{1}{6} \quad Pr(5) = \frac{1}{6}$$

$$Pr(4 \text{ then } 5) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

change the experiment - rolling two dice!

die #1 = $\{1, 2, 3, 4, 5, 6\}$

die #2 = $\{1, 2, 3, 4, 5, 6\}$

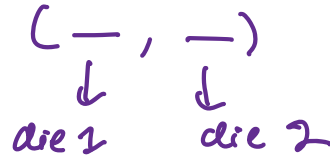
$$S = \{(1,1), (1,2), \dots, (1,6), (2,1), (2,2), \dots, (2,6), \dots, (6,1), (6,2), \dots, (6,6)\}$$

what is sample space?

a#1 x a#2 set of ordered pairs

Cartesian product!

$|S| = 36$ ordered pairs



using this sample space, what are probabilities in craps?

was Larry right?
 more likely to lose == rigged!

outcome	Prob
2, 3, 12 - lose !!	4/36
7, 11 - win !!	8/36
anything else - draw	24/36

Event Space

$$\{(1,1), (1,2), (2,1), (6,6)\}$$

$$\{(1,6), (6,1), (3,4), (4,5),$$

$$(5,2), (2,5), (5,6), (6,5)\}$$

↳ **Subtraction rule!** $36 - (8 + 4) = 24$

Example w/ birth months → pigeonhole = guaranteed
 probability = likely, unlikely

• assume = all months equally likely

one random person

• $Pr(\text{Oct}) = 1/12$

• $Pr(\text{non-Oct}) = 11/12$

10 random people

• $Pr(\text{everyone in Oct}) = (1/12)^{10}$

product rule!

• $Pr(\text{none in Oct}) = (11/12)^{10} = .4189$

• $Pr(\text{at least one in Oct}) = 1 - .4189 = .58$

1 - invalid cases

subtraction rule!

(not "at least one in Oct")

(= none in Oct)

ex) hand of poker (5 cards)

• # 5 card hands $(52, 5)$

• # hands w/ pair

- pair 13 values
 $(4, 2)$ suits

- non pair $(12, 3)$ values
4 suits

together $13 \cdot 6 \cdot 220 \cdot 4^3$

order doesn't matter! $Pr(\text{pair}) = \frac{\# \text{ pairs}}{\# 5 \text{ card}}$

$= \frac{1.098 \text{ mil}}{2.5 \text{ mil}} = .4225 = 42.25\%$

ex) draw 2 cards from deck
no replacement

order matters

$|S| = 52 \cdot 51$

$|E| = 13 \cdot 12$

$Pr(\text{2 hearts}) = \frac{13 \cdot 12}{52 \cdot 51} = .0588$

order doesn't matter

$|S| = (52, 2)$

$|E| = (13, 2)$

$Pr(\text{two hearts}) = \frac{(13, 2)}{(52, 2)} = .0588$

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omg!