CS1800
10/17 - Fri :)

Admin

- HW4 due 10/21 9pm
- Quiz 2 next Fri in class

Agenda

1. Pigeonhole Principle
2. Probability

lecture question

The quiz 2 content says sets and set operations as well as counting. What is the counting content cut off? Will permutations combinations and stars and bars be included?

↳ yes! HW3, HW4

sets, set operations, set equality

product, sum, combos, perms, stars + bars, overcounting

1. Pigeonhole Principle

counting ⟷ probability

guarantees! (prob = likely, unlikely)

boxes ☐

objects

"have to" "must" "at least"

ceiling $\lceil \rceil$ round up to next integer

$\lceil 2.7 \rceil = 3$

$\lceil 2.1 \rceil = 3$

$\lceil 2 \rceil = 2$

$\lceil 2.0001 \rceil = 3$

joinpd.com <slides>

2. Probability → sets, counting

experiment: infinitely repeatable procedure with a well-defined set of outcomes

sample space S: all possible outcomes of an experiment

$S = \{s_1, s_2, \ldots, s_n\}$ s_i = one outcome

event space E: $E \subseteq S$ subset of outcomes we care about

$E_1 = \{s_1\}$ $E_2 = \{s_2, s_3\}$ $E_3 = \{s_1, s_3, s_5\}$

probability of event E $Pr(E) = \frac{|E|}{|S|}$ cardinality

ex 6-sided die  experiment = rolling the die

"fair" – all outcomes equally likely

$S = \{1, 2, 3, 4, 5, 6\}$

event examples

$E = \{3\}$ $Pr(E) = \frac{|E|}{|S|} = \frac{1}{6}$

$E = \{1, 3, 5\}$ $Pr(\text{odd}) = \frac{|E|}{|S|} = \frac{3}{6} = \frac{1}{2} = .5$

↳ roll a 1, 3, or 5 simple!

$Pr(1) + Pr(3) + Pr(5) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

ex roll the die twice

$Pr(4 \text{ then } 5)$? product rule!

$Pr(4) = \frac{1}{6}$ $Pr(5) = \frac{1}{6}$

$Pr(4 \text{ then } 5) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

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change the experiment – rolling two dice!

die #1 = {1, 2, 3, 4, 5, 6}

die #2 = {1, 2, 3, 4, 5, 6}

what is sample space? cartesian product!

d#1 x d#2 set of ordered pairs

$S = \{(1,1), (1,2), \ldots (1,6),$
$(2,1), (2,2), \ldots, (2,6),$
$\vdots$
$(6,1), (6,2) \ldots (6,6)\}$

$|S| = 36$ ordered pairs

$(\_, \_)$
↓ ↓
die 1 die 2

using this sample space, what are probabilities in craps?

outcome	Prob
2, 3, 12 – lose :(	4/36
7, 11 – win :)	8/36
anything else – draw	24/36

↳ subtraction rule $36 - (8+4) = 24$

was Larry right? more likely to lose == rigged!

Event space

$\{(1,1), (1,2), (2,1), (6,6)\}$

$\{(1,6), (6,1), (3,4), (4,3),$
$(5,2), (2,5), (5,6), (6,5)\}$

example w/ birth months → pigeonhole = guaranteed

probability = likely, unlikely

- assume = all months equally likely

one random person
- $Pr(\text{Oct}) = 1/12$
- $Pr(\text{non-Oct}) = 11/12$

10 random people
- $Pr(\text{everyone in Oct}) = \left(\frac{1}{12}\right)^{10}$

product rule!

- $Pr(\text{none in Oct}) = \left(\frac{11}{12}\right)^{10} = .4189$
- $Pr(\text{at least one in Oct}) = 1 - .4189 = .58$

$1 -$ invalid cases (not "at least one in Oct") (= none in Oct)

subtraction rule!

(ex) hand of poker (5 cards)
- # 5 card hands $C(52,5)$
- # hands w/ pair
  - pair: 13 values, $C(4,2)$ suits
  - non pair: $C(12,3)$ values, 4^3 suits

together $13 \cdot 6 \cdot 220 \cdot 4^3$

order doesn't matter! $Pr(\text{pair}) = \frac{\#\text{ pairs}}{\#\text{ 5 card}}$

$= \frac{1.098 \text{ mil}}{2.5 \text{ mil}} = .4225 = 42.25\%$

(ex) draw 2 cards from deck, no replacement

$Pr(\text{two hearts})$?

order matters

$|S| = 52 \cdot 51$

$|E| = 13 \cdot 12$

$Pr(\text{2 hearts}) = \frac{13 \cdot 12}{52 \cdot 51} = .0588$

order doesn't matter

$|S| = C(52,2)$

$|E| = C(13,2)$

$Pr(\text{two hearts}) = \frac{C(13,2)}{C(52,2)} = .0588$

omg!