CS1800
9/16-Tues.

Admin

- HW1 is out! :)
  - ↳ due 9/23 9pm
  - ↳ 48-hour grace period
  - ↳ 2nd chance hw (1-6)
- rec 2 this week to prep for hw
- say your name!

Agenda

1. Implication versions
2. Predicates + Quantifiers
3. negating predicates

0. Friday Catch-up

goooo

P = It's raining
Q = Laney takes the T

$P \Rightarrow Q$
Laney takes the T if it rains
False once: rain, no T

$\neg(P \Rightarrow Q)$
True once: rain, no T

English? $\neg(P \Rightarrow Q)$
raining, but don't take the T

$\neg(P \Rightarrow Q)$
$\neg(\neg P \vee Q)$ def of imp
$\neg\neg P \wedge \neg Q$ demorg
$P \wedge \neg Q$ double neg

Laney takes the T only if it rains
False once: T, but no rain

T if it rains
T only if it rains
↳ converse

same meaning? $\boxed{\text{no!}}$

1. Implication versions

Home base $P \Rightarrow Q$

		$P \Rightarrow Q$
$\neg(P \Rightarrow Q)$	negation	$\not\equiv$
$Q \Rightarrow P$	converse	$\not\equiv$
$\neg P \Rightarrow \neg Q$	inverse	$\not\equiv$
$\neg Q \Rightarrow \neg P$	contrapositive	$\equiv$

want to prove $P \Rightarrow Q$
- sometimes, easier to prove contra
- converse, inverse can sound similar but proving them doesn't help

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow Q$	$\neg P \Rightarrow \neg Q$	$\neg Q \Rightarrow \neg P$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	T

If we prove the contrapos, we prove the original

ex $P = n^2$ is not divisible by 3
$Q = n$ is not divisible by 3

domain: integers

goal $P \Rightarrow Q$

If n^2 is divisible by 3, then n is divisible by 3

inverse

If n is not divisible by 3, then n^2 is not divisible by 3

converse

If n is divisible by 3, then n^2 is divisible by 3

contra! :)

- divisible by 3

$3 \cdot$ ___
$3 \cdot 1 = 3$
$3 \cdot 2 = 6$
$3 \cdot 3 = 9$
...

If n is divis by 3, then n^2 is divis by 3

↳ assumption, just true! } given this assumption, show n^2 is divis by 3

$n = 3k$ for some integer k

$n^2 = n \cdot n$
$= 3k \cdot 3k$
$= 3 \cdot 3 \cdot k^2$

so, $n^2 = 3 \cdot$ ___
and is a multiple of 3

2. Predicates + quantifiers

↳ slides

3. negating predicates

practice with predicates

$j(x)$ johnny fights x $k(x)$ x karates
Domain: cobra kai characters

- English to predicate logic

The only people Johnny fights are people who study karate.

$\forall x \ \text{johnny}(x) \Rightarrow \text{karate}(x)$

From slides: johnny fights everyone who studies karate

$\text{karate}(x) \Rightarrow \text{johnny}(x)$

J	K	J⇒K	K⇒J
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

↳ I see you in the dojo, you def go and fight Johnny

↳ I see J fighting you, you are def a karate student

- Negation: negate $\exists x \ k(x) \wedge j(x)$

$\neg(\exists x \ \text{karate}(x) \wedge \text{johnny}(x))$

$\forall x \ \neg(\text{karate}(x) \wedge \text{johnny}(x))$

$\forall x \ \neg\text{karate}(x) \vee \neg\text{johnny}(x)$

- same subtle difference as T/rain, if/only if from v. beginning of today!

Quantifier examples. Domain = integers

(a) $\forall x \exists y (x + y = 0)$ For all x, there is a y s.t. $x+y=0$ True!

(b) $\exists y \forall x (x + y = x)$ There exists y s.t. for all x, $x+y=x$ True! $y=0$

(c) $\exists x \forall y (x + y = x)$ There exists x s.t. for all y, $x+y=x$ False!