CS1800 Fall 2025

Revisit & Practice: How many ways... ?

Where to start...?

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	Repetition NOT OK	Repetition OK
Order Matters	$P(n,k) = \frac{n!}{(n-k)!}$ $n \cdot n-1 \cdot n-2 \cdot n-3$	n^k $n \cdot n \cdot n \cdot n$
Order Doesn't Matter	$C(n,k) = \frac{n!}{k!\,(n-k)!}$ A, B = B, A	$C(n,k)$ where n = #stars + #bars k = #stars

Where to start...?

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There are 25 players on the Red Sox roster. How many ways to choose 9 starters, if anyone could play any position?

- rep not ok ~ can't choose same person twice
- order doesn't matter ~ give me 9 players

$C(n,k)$ $n=25$, $k=9$

$$\frac{25!}{9!\,16!} = 2,042,975$$

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There are 25 players on the Red Sox roster. How many ways to choose someone for first, second, and third-base?

- rep not ok – one person can't play multiple positions
- order matters – labels on positions

1st	Lowrey	$\neq$	1st	Varitek
2nd	Story		2nd	Lowrey
3rd	Varitek		3rd	Story

$P(n,k)$ $n=25$

$k=3$

$$\frac{25!}{22!} = 25 \cdot 24 \cdot 23 = 13{,}800$$

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You have 5 pink socks and 8 orange socks. How many ways to pick two socks?

- order doesn't matter — choosing O, P = P, O
- rep ok — pink sock = every other pink sock
  if I get 2 pink socks, 2 of the same thing

C(n, k) with n = # stars + # bars
k = # stars

stars = 2   # bars = 1

$C(n,k) = C(3,2) = \frac{3!}{2!\,1!} = 3$

pink | orange

* * |

* | *

| * *

How many bit strings of length 3 with exactly 2 zeroes?

How many ways to assign 4 of the 6 CS1800/1802 instructors to 4 different classrooms?

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How many ways to assign 4 of the 6 CS1800/1802 instructors to 4 different classrooms?

different == we can tell them apart

- order matters ~ $\underset{1}{\underline{6}} \cdot \underset{2}{\underline{5}} \cdot \underset{3}{\underline{4}} \cdot \underset{4}{\underline{3}}$
- rep not ok
  - ↳ can't repeat people / rooms

$P(n,k)$ $n=6$, $k=4$

$$\frac{6!}{2!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = 6\cdot5\cdot4\cdot3 = 360$$

How many ways to pick 4 of the 6 CS1800/1802 instructors to get a raise?

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How many ways to pick 4 of the 6 CS1800/1802 instructors to get a raise?

- order doesn't matter   A, B, C, D = D, C, B, A
- rep not ok - can't choose same person twice

$C(n, k)$ $n = 6$, $k = 4$

$$\frac{6!}{2!\,4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} = \frac{6 \cdot 5}{2} = 15$$

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How many five-card hands exist?

- rep not ok - can't have same card twice
- order doesn't matter

$C(n,k)$ $n=52$ $k=5$ $C(52,5) = 2,598,960$

There are 52 cards in a deck. 13 values (Ace, 2, 3, 4, ..., 10, Jack, Queen, King) each in four suits (♣ ♦ ♥ ♠)

How many five-card hands have a pair (2 cards the same value; no other pairs/matching)?

How many five-card hands have three-of-a-kind (3 cards the same value; no other pairs/matching)?

the 3 cards and the remaining 2 cards

- value $C(13,1)$
- suits $C(4,3)$

- values $C(12,2)$
- suits $C(4,1)$ 2 times

Together: $C(13,1) \cdot C(4,3) \cdot C(12,2) \cdot 4^2$

$= 54,912$

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How many five-card hands have a pair (2 cards the same value; no other pairs/matching)? ↳ product rule pair and non-pairs

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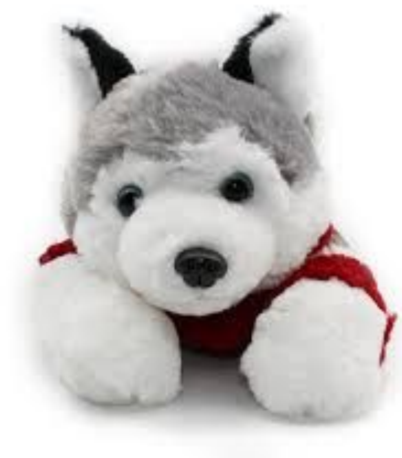
- How many ways to have a pair?
  - value $C(13,1) = 13$
  - suits $C(4,2) = 6$

  $13 \cdot 6$

- How many ways to have the last 3 cards?
  - values $C(12,3)$
  - suits $C(4,1) \cdot C(4,1) \cdot C(4,1) = 4^3$

All together: $13 \cdot 6 \cdot C(12,3) \cdot 4^3 = 1,098,240$

There are 25 Husky stuffies and 3 shelves at the bookstore. How many ways to arrange the stuffies on the shelves?



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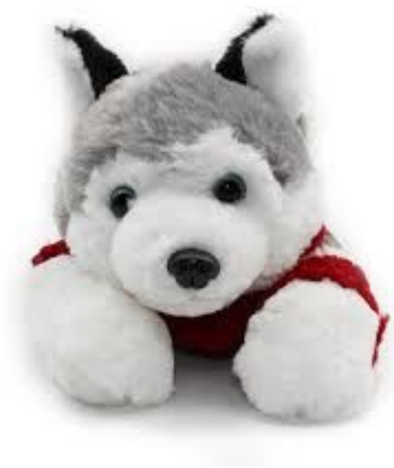

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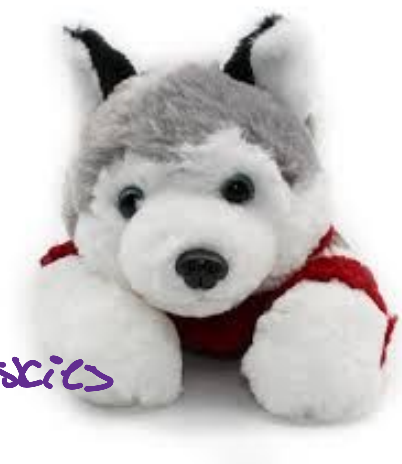

- rep ok
- order doesn't matter

shelf 1 | shelf 2 | shelf 3

#huskies | #huskies | #huskies

Huskies are indistinguishable

$C(n, k) = (27, 25) = 351$

#stars 25

#bars 2

How many ways to split up 10 students into Sec1 with 5 people and Sec 2 with 5 people?

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product rule with...

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How many ways to split up 10 students into Sec1 with 5 people and Sec 2 with 5 people?

___ · ___ product rule

Sec 1   Sec 2

within a section, order doesn't matter

sec 1: ABCDE = ECDBA

sec 2: fghij = jihfg

sec 1: $C(10,5)$

sec 2: $C(5,5) = 1$

$C(10,5) \cdot C(5,5) = 252$

How many ways to split up 10 students into two groups of 5?

How many ways to split up 10 students into two groups of 5?

- within a group, order doesn't matter

C(10,5)

C(5,5)

why is this different?

↳ groups now have no labels, indistinguishable

C(10,5) · C(5,5) assumes groups are distinguishable

↳ sec 1: ABCDE, sec 2: LMNOP ≠ sec 1: LMNOP, sec 2: ABCDE

What we want: ABCDE, LMNOP one outcome

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How many ways to arrange the letters *N O R T H*?

- order matters   north ≠ thorn
  "arrange"
- rep not ok   can't repeat a letter

n o r t h

t t t r r × not ok! !!

$P(5,5) = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

How many ways to arrange the letters *E A S T E R N*?

How many ways to arrange the letters *E A S T E R N*?

north $P(5,5) = 5!$

eastern is not $P(7,7) = 7!$

↳ Because there are 2 e's they are indistinguishable

$e_1\, a\, s\, t\, e_2\, r\, n \neq e_2\, a\, s\, t\, e_1\, r\, n$

→ 7! says that these are 2 different outcomes

why is this different?