CS1800
9/12 - Fri :)

Admin
- Pizza?
- HW1 out next Thurs.
- Say your name!

Agenda
1. Logical Equivalence
2. wait, are they equivalent?
3. negation + implications

0. Review!

P	Q	$\neg P$	$\neg Q$	$P \land Q$	$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$	$\neg(P \land Q)$	$\neg P \lor \neg Q$
T	T	F	F	T	T	T	F	F
T	F	F	T	F	F	F	T	T
F	T	T	F	F	T	T	T	T
F	F	T	T	F	T	T	T	T

$\neg Q$	$\neg P$
F	F
T	F
F	T
T	T

A = it's night B = it's dark

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

If it's night, then it's dark
$A \Rightarrow B$

1. Logical Equivalence

If 2 statements have same truth values for all inputs, then they are logically equivalent $\equiv$

Proved above! $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

$\neg(P \land Q) \equiv \neg P \lor \neg Q$ ☆

Proof Techniques:
1. Truth tables ☑
2. Logical equivalence laws
3. Proof by counterexample

...

All proofs:
- convince someone else
- clear
- take small steps
- narrative

ex truth table proof

P	Q	$\neg P$	$\neg Q$	$P \lor Q$	$\neg(P \lor Q)$	$\neg P \land \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

and therefore, ...

$\neg(P \lor Q) \equiv \neg P \land \neg Q$ ☆ DeMorgan's Law!

Show equivalence using logical equivalence laws:
- one law at a time
- label each step

Logical Equivalence Laws

Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$ *
Domination laws	$p \wedge F \equiv F$ $p \vee T \equiv T$ *
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$ * proved!
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Double negation	$\neg(\neg p) \equiv p$ *
Absorption	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$

def of implication: $P \Rightarrow Q \equiv \neg P \vee Q$

double negation: $P \equiv \neg(\neg P)$

P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

identity law: $P \wedge T \equiv P$

P	T	$P \wedge T$
T	T	T
F	T	F

domination! $P \vee T \equiv T$

P	T	$P \vee T$
T	T	T
F	T	T

ex $\neg(\neg p \vee q) \wedge T \equiv p \wedge \neg q$

- start with LHS, make it look like RHS

$\neg(\neg p \vee q) \wedge T$

$(\neg\neg p \wedge \neg q) \wedge T$ #demorgan

$(p \wedge \neg q) \wedge T$ #double neg

$p \wedge \neg q$ #identity

done!

ex $p \Rightarrow (p \wedge q) \equiv (\neg p \vee q)$

$p \Rightarrow (p \wedge q)$

$\neg p \vee (p \wedge q)$ #def of imp.

$(\neg p \vee p) \wedge (\neg p \vee q)$ # distro

$T \wedge (\neg p \vee q)$ #negation

$\neg p \vee q$ #identity done!

2. wait, are they equivalent?

↳ logical equivalence proof

1. show equivalence ☑
2. simplify ☑ → given one logic statement make as simple as possible (reduce # of operators, variables)
3. show equivalence or inequivalence ☑
   ↳ show with equivalence laws or counterexample, assign values to P, Q

ex $(p \wedge q) \Rightarrow p$

$\neg(p \wedge q) \vee p$ # def of imp.

$(\neg p \vee \neg q) \vee p$ # demorg

$\neg p \vee \neg q \vee p$ # assoc.

$p \vee \neg p \vee \neg q$ # comm.

$T \vee \neg q$ # negation

T # domination! ing

ex which one is correct? Prove it

$p \vee \neg(p \wedge q) \equiv T$?

$p \vee \neg p \vee \neg q$ #demorg

$T \vee \neg q$ # neg

T # domination

$p \vee \neg(p \wedge q) \equiv F$?

- show inequivalence
- counterexample
- set P, Q to truth values such that

  LHS $\not\equiv$ RHS
- plug values in, simplify

P = True, Q = False RHS = F

$T \vee \neg(T \wedge F)$

$T \vee \neg F$

$T \vee T$

T x

Incorrect, counterexample!

ex which is correct? prove it

$p \wedge (q \vee \neg q) \equiv q$?

- not equivalent!
- counterexample
- assign values to P, Q such that

  LHS $\not\equiv$ RHS

P = True Q = False RHS = False

plug in:

$T \wedge (F \vee \neg F)$

$T \wedge (F \vee T)$

$T \wedge (T)$

True

done!

$p \wedge (q \vee \neg q) \equiv p$?

- equivalent!
- apply laws

$p \wedge (q \vee \neg q)$

$p \wedge T$ #negation

p #identity

3. Negation + implication

How do you intuitively know what operators to use and what order to put the P, Q, R, S, statements in when you are transforming english to logic? Can you through an example of your thought process?

ex P = it's raining
Q = Laney takes the T

$P \wedge Q$ rain and the T
the T and rain
rain, but the T
rain or the T

both things are true

$P \Rightarrow Q$

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

If rain, then the T
rain, so the T
if rain, then the T
when it rains, the T
rain, therefore the T
the T because it's raining

negation of $P \Rightarrow Q$?

$\neg(P \Rightarrow Q)$

$\neg(\neg P \vee Q)$ def of imp

$\neg\neg P \wedge \neg Q$ demorg

$P \wedge \neg Q$ double negation

in English?

I take the T if it rains

I take the T only if rain

see Laney on red line
what can you conclude?

$\boxed{\text{think about for Tuesday!}}$