CS1800
10/10 - Fri !! :)

Admin

- HW3 due 10/14 9pm
- no OH monday

Agenda

1. Product / Permutations
2. Sum Rule / Combinations
3. Stars + Bars

lecture questions

what do disjoint sets mean?

$A \cap B = \{\}$ A, B are disjoint

In the password restriction question, the one we did towards the end of the lecture - How did we get the 10 * 36^4 for the passwords which start with a number?

- pwd of length 5
- alphanumeric
- repeats ok

How many start w/ #?

$\underline{\#}\ \underline{\text{any}}\ \underline{\text{any}}\ \underline{\text{any}}\ \underline{\text{any}}$

$10 \cdot 36 \cdot 36 \cdot 36 \cdot 36$

$10 \cdot 36^4$

0. Review

		sols	rep ok?
and	multiply	3^3	✓
or	add	$3 \cdot 2$	X
		$3 \cdot 2 + 2 \cdot 3$	?

(one thing from 2 different sets)

1. Product / Permutations

→ context

	no rep	rep ok
order matters	permutation $P(n,k)$	n^k
order doesn't matter	$C(n,k)$	stars + bars $*$ | (combination)

→ product rule!

when order matters, generally ... product rule

$\underline{1}$ $\underline{2}$ → multiply
#choices #choices

when rep ok ... k tasks
n choices per task

n^k

when rep not ok ... k tasks
choices go down: $n, n-1, n-2, \ldots$

Permutation:

k permutation of n objects is an ordering of k of the objects

$P(n,k)$

ex 535 ppl run, ways to have 1st, 2nd, 3rd

product rule: $\underset{535}{\underline{1^{st}}}$ $\underset{534}{\underline{2^{nd}}}$ $\underset{533}{\underline{3^{rd}}}$

$= 535 \cdot 534 \cdot 533$

ex Laney has 4 donuts, starting in a set
$\{P, G, C, B\}$ → order in which I eat them?

$n=4$, $k=1,2,3,4$

$k=4$ (all donuts)

$\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 24$

$4!$

$k=3$

$\underline{4} \cdot \underline{3} \cdot \underline{2} = 24$

$4!$

$k=2$

$\underline{4} \cdot \underline{3} = 12$

$\frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$

$k=1$

$\underline{4} = 4$

$\frac{4!}{3!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$

$P(n,k) = P^n_k = \frac{n!}{(n-k)!}$

numerator: all possibilities
denom: when to stop multiplying

$P(n,n) = n!$

examples

7 ppl, taking a pic. Arrange...
- all of them? $P(7,7) = 7! = 5040$
- 5 of them? $P(7,5) = \frac{7!}{2!} = 2520$
- 2 of them? $P(7,2) = \frac{7!}{5!} = 7 \cdot 6 = 42$

How many words don't repeat letters, of length...
- 1? 26
- 2? $P(26,2) = 26 \cdot 25 = 650$
- 5? $P(26,5) = \frac{26!}{21!} = 7.8m$

100 ppl audition. Cast...
- Mac? 100
- Mac, Lady? $100 \cdot 99$
- Mac, Lady, McD? $100 \cdot 99 \cdot 98$

notice... what makes these permutations? (order matters, no rep)
- ppl/things can't be repeated
  - one person can't be two roles, in same photo twice
- labels == order matters

- "arrange" order matters
- "can't repeat" no rep
- words $IT \neq TI$

2. Sum Rule / Combination

any time order doesn't matter: combination
- no rep $C(n,k)$
- rep ok stars+bars

$C(n,k) = C^n_k = \binom{n}{k}$

k-Combination: selecting k objects from a set of n objects

selecting, choosing $\neq$ arranging, ordering

ex simplification of password ~ bit string: string of 0/1 $001 \neq 01$
don't care about numeric value

1. How many bit strings of length 5?
product rule, rep ok $\frac{\_}{2}\,\frac{\_}{2}\,\frac{\_}{2}\,\frac{\_}{2}\,\frac{\_}{2} = 2^5 = 32$

2. How many of length 5 start with 1?
$\frac{1}{1}\,\frac{\_}{2}\,\frac{\_}{2}\,\frac{\_}{2}\,\frac{\_}{2} = 2^4 = 16$

3. How many of length 5 have exactly one 1?

10000
or 01000
or 00100
or 00010
or 00001

sum rule: add cases
$1+1+1+1+1=5$

better: positions in a set $\{1,2,3,4,5\}$
choose 1

4. How many have exactly 2 1s?
positions in a set $\{1,2,3,4,5\}$
choose 2
order doesn't matter 34 = 4,3

$C(n,k)$ with $n=5, k=1$

no repetition : one pos per bit

$$\boxed{C(n,k) = \frac{n!}{k!(n-k)!}}$$

$$C(5,1) = \frac{5!}{1!\,4!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{1\cdot4\cdot3\cdot2\cdot1} = 5$$

10010
01100
10100
10001
...

$C(n,k)$ with $n=5$, $k=2$

$$\frac{5!}{2!\,3!} = \frac{5\cdot4}{2} = 10$$

examples

7 ppl, say hello...

- to everyone? $C(7,7)=1$
- to 5 ppl? $C(7,5) = \frac{7!}{2!\,5!} = 21$
- to 2 ppl? $C(7,2) = \frac{7!}{5!\,2!} = 21$

choose letters on wheel?

- 1 letter? 26
- 2 letters? $C(26,2) = \frac{26!}{2!\,24!} = \frac{26\cdot25}{2}$
- 5 letters? $C(26,5) = \frac{26!}{5!\,21!}$

100 ppl audition. Cast...

- 3 witches? $C(100,3) = \frac{100!}{3!\,97!} = \frac{100\cdot99\cdot98}{3!}$

what do we notice about $C(n,k)$... order doesn't matter, no rep

- no titles/labels = order doesn't matter
  - ↳ cast A, B, C as the witches same as B, C, A, C, A, B, etc.
- $C(7,5) = C(7,2)$ ... $C(n,k) = C(n, n-k)$
- permutations get smaller as k gets smaller $P(n,n) > P(n,1)$ ...
- combos start small, get big, get small $C(n,n)=1$, $C(n,1)=n$
- combo < perm

  2 letters: $\frac{26\cdot25}{2}$ vs. $26\cdot25$

  IT = TI order doesn't matter

  IT ≠ TI order matters

3. Stars + Bars

↳ order doesn't matter, repetition ok > def. a combination! $C(n,k)$ what's n?

bit strings ⇝ 0 = *, 1 = |

How many strings of length 5 have exactly 2 bars?

|**|*
*|**|
***||

$$C(5,2) = \frac{5!}{2!\,3!} = 10$$

ex) donuts at Dunkin to buy them
- order doesn't matter (toss in a bag)
- rep ok (> 1 donut of same flavor)

↳ same flavor = indistinguishable

↳ 3 flavors P, G, C. I want 3 donuts.

* donuts | separate flavors

P	G	C
	**	*
*	**	
***		
*	*	*

$C(n,k)$ with $n = \#\text{stars} + \#\text{bars}$

$k = \#\text{stars}$

$n = 3+2 = 5 \quad k = 3 \qquad C(5,3) = C(5,2) = 10$:)

$C(5,3) \to \frac{5!}{3!\,2!} \qquad C(5,2) \to \frac{5!}{2!\,3!}$

\#bars = \#cats − 1 (dividers)

\#stars = selections

$$C(26,2) = \frac{26!}{2!\,24!} \qquad \frac{26 \cdot 25 \cdot \cancel{24 \cdot 23 \cdot 22 \cdots 1}}{(2 \cdot 1)\cancel{(24 \cdot 23 \cdot 22 \cdots 1)}} \qquad \frac{26 \cdot 25}{2}$$

A

ACDE B

AB DF C

ABC E D

ABCD E