

CS1800

Fall 2025

Homework 7

Assigned: November 18, 2025

Deadline: December 2, 2025 at 9pm eastern <==== extra time b/c of holiday! :)

Late Deadline: December 4, 2025 at 9pm eastern

Submission Format

CS1800 Homeworks must be neatly handwritten, on physical paper or a tablet. **Typed submissions will not be accepted.**

Submit a PDF or images to Gradescope. If you submit images, make sure they are JPEG, JPG, or PNG. (Don't submit iPhone HEIC images!) Each problem starts on a new page. Problem 1 is on a different page than Problem 2. Problems 1A and 1B can be on the same page.

Collaboration and Academic Integrity

We encourage you to work with classmates on these problems; list all collaborators on the first page of your submission. You may also consult outside sources, including generative AI; list all outside sources you referenced and how you used them on the first page of your submission.

However, **you must write all your solutions yourself, in your own words.** Do not submit anything you can't explain. Copying solutions from another person or an outside source is a violation of our academic integrity policy.

Deadline and Late Submissions

You can submit any/all homeworks up to 48 hours late with no penalty. However, **once the late deadline has passed, no submissions will be accepted.** This policy exists so you take extra time when you're especially busy, not feeling well, tending to your family, etc.; we won't make any exceptions to this policy.

Second-Chance Homework: You will have an opportunity at the end of the semester to submit one homework (HWs 1-6 only) for a new grade. If you miss both the deadline *and* the late deadline on a homework, use the Second-Chance to submit it.

For full credit

- select which pages go with which questions on Gradescope, and
- show ALL your work including intermediate steps.

Your Name

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Your Collaborators (Classmates)

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Outside References and How You Used Them

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For all mathematical induction proofs, all four components (predicate -- usually provided, logic statement, base case, and inductive step) must be included and clearly labelled. Any application of the Inductive Hypothesis must be clearly noted. Take small steps in your proof, and remember that you are convincing the reader!

Problem #1 - Proof by Induction - Summations (8 points)
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Use mathematical induction to show the following summation:

Predicate: $P(x)$ states that $\sum_{i=1}^x i^2 = \frac{x(x+1)(2x+1)}{6}$

Problem #2 - Proof by Induction - Inequality (8 points)

When we cover the growth of functions toward the end of the semester, we'll use the assumption that quadratic functions grow slower than exponential functions, i.e., that $n^2 < 2^n$. Find the smallest positive integer for which this is true, and will always be true for that value and higher.

Predicate $P(x)$ states that $x^2 < 2^x$

Problem #3 - Proof by Induction - Divisibility (8 points)

Use mathematical induction to show the following predicate is true for all positive integers.

Predicate: $P(x)$ states that $(x^5 - x)$ is a multiple of 5.

Problem #4 - Proof by Induction - Data Structures (8 points)

Use Mathematical Induction to prove the following property about the cardinality of powersets.

Predicate $P(x)$ states that, for an arbitrary set A with x elements, $|\mathcal{P}(A)| = 2^x$.