

## fCS1800

Fall 2025

### Homework 4

**Assigned:** October 14, 2025

**Deadline:** October 21, 2025 at 9pm eastern

**Late Deadline:** October 23, 2025 at 9pm eastern

#### Submission Format

CS1800 Homeworks must be neatly handwritten, on physical paper or a tablet. **Typed submissions will not be accepted.**

Submit a PDF or images to Gradescope. If you submit images, make sure they are JPEG, JPG, or PNG. (Don't submit iPhone HEIC images!) Each problem starts on a new page. Problem 1 is on a different page than Problem 2. Problems 1A and 1B can be on the same page.

#### Collaboration and Academic Integrity

We encourage you to work with classmates on these problems; list all collaborators on the first page of your submission. You may also consult outside sources, including generative AI; list all outside sources you referenced and how you used them on the first page of your submission.

However, **you must write all your solutions yourself, in your own words.** Do not submit anything you can't explain. Copying solutions from another person or an outside source is a violation of our academic integrity policy.

#### Deadline and Late Submissions

You can submit any/all homeworks up to 48 hours late with no penalty. However, **once the late deadline has passed, no submissions will be accepted.** This policy exists so you take extra time when you're especially busy, not feeling well, tending to your family, etc.; we won't make any exceptions to this policy.

Second-Chance Homework: You will have an opportunity at the end of the semester to submit one homework (HWs 1-6 only) for a new grade. If you miss both the deadline *and* the late deadline on a homework, use the Second-Chance to submit it.

#### For full credit

- select which pages go with which questions on Gradescope, and
- show ALL your work including intermediate steps.

**Your Name**

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**Your Collaborators (Classmates)**

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**Outside References and How You Used Them**

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**Problem #1 - Counting (Focus on Permutations, Combinations) ( $3+3+3+3=12$  points)**

Laney's husband Tom (he/him) works at a food bank and often makes deliveries to local homes. He has 10 unique boxes of food to deliver today.

Give each answer as a single number and show your work.

**1A** How many ways can Tom select 6 boxes to deliver?

**1B** How many different routes can Tom drive to deliver 8 of the 10 boxes? A route is an ordered sequence of destinations. You may assume each package may only be delivered to its distinct destination.

**1C** Before leaving for the day, Tom loads leftover boxes into the storage room. How many ways can 8 distinct boxes be loaded into 4 shelves where each shelf can hold up to 8 boxes? (The ordering on a shelf doesn't matter.)

**1D** How many ways can 8 distinct boxes be loaded into 4 shelves where each shelf must have the same number of boxes?

**Problem #2 - Counting (Focus on Stars and Bars)** ( $3+3+3=9$  points)

Laney invites Prof. Patterson, Prof. Sloan, and Prof. Sanghrajka over for her birthday (March 27th, btw). Prof. Sanghrajka bakes a vegan cake, which is delicious, and cuts it into 8 pieces.

- 2A** How many ways are there to distribute the 8 pieces among the group, if Laney wants to make sure that Prof. Patterson doesn't get any?

- 2B** How many ways are there to distribute the 8 pieces among the three guests (still assuming Prof. Patterson doesn't get any), such that Laney, personally, gets at least 4 pieces?

- 2C** Assume that each person can eat a maximum of 5 pieces. Given this limitation, and still not giving any cake to Prof. Patterson, how many ways are there to distribute the 8 pieces among the three guests?

**Problem #3 - Counting (Focus on: Permutations, Combinations, Overcounting)**

*(3+3+3+3=12 points)*

Khoury College is forming a trendy pickleball team. They've found 10 people who play pickleball in Jamaica Plain, and 14 in nearby, superior Dorchester. The coach, Coach Matt O'Shea from Dorchester, wants to pick people for his team.

Show all of your work, including specifying separate cases where applicable. Report your final answer as an expression such as  $C_k^n$  or  $n^k$ .

- 3A** Coach Matt splits all 24 players into two teams of 12 each, for a scrimmage. He gives the Blue Team blue pinnies and the Red Team red pinnies. How many ways can Coach Matt split the players into a Red Team and a Blue Team?

- 3B** Now the scrimmage is over and it's time for evaluation. Coach Matt throws away the pinnies, and splits all 24 players into three teams of 8 each, so he can evaluate the best ones. Two teams are the same if they have the same players. How many ways can Coach Matt split the players into three teams?

**3C** Each of the 24 players is either a Leftie or a Rightie. Coach Matt selects 5 Lefties and 8 Righties and lines them up on the baseline. How many ways are there to arrange this group, assuming Lefties are indistinguishable from each other and so are Righties?

**3D** Coach Matt wants to have fair representation from the two best neighborhoods in Boston. How many ways can he choose 9 total pickleballers for his team that includes at least 3 players from Dorchester (best neighborhood) and at least 3 players from Jamaica Plain (distant second-best)?

**Problem #4 - Counting (General)** ( $3+3+3+3=12$  points)

For each subpart below, give your solution as a whole number.

We are going trick-or-treating 🎃. 8 people have dressed up as ghosts ( $g_1, g_2, \dots, g_8$ ) and 8 people have dressed up as vampires ( $v_1, v_2, \dots, v_8$ ). (Even though they have similar costumes, we can tell people apart!)

**4A** Six of the vampires line up side-by-side for a picture. How many ways to arrange them?

**4B** Two of the vampires,  $v_1$  and  $v_2$  are dressed up like Spike and Druscilla and insist on always standing next to each other (in either order) for a picture. How many ways to arrange the 8 vampires under this constraint?

**4C** How many ways to arrange 3 of the vampires and 2 of the ghosts for the picture such that no two vampires sit next to each other (e.g., one acceptable arrangement would be  $v_1, d_1, v_2, d_2, v_3$ )?

**4D** All 8 ghosts decide to leave the photo session and haunt the dining room by sitting around a round table. We'll consider two arrangements the same if everyone has the same left and right ghost neighbors (e.g.,  $g1, g2, g3$  is the same as  $g2, g3, g1$  but not the same as  $g3, g2, g1$ ). How many ways to arrange the ghosts?

**Problem #5 - Counting (General)** (3+3+3=9 points)

Recall that a deck of cards comprises 52 total cards, each with a suit and a value. There are four suits ( $\clubsuit$   $\diamondsuit$   $\heartsuit$   $\spadesuit$ ), and 13 values of each suit (2, 3, 4, ..., 10, J, Q, K, A).

In Poker, a *straight* is a hand consisting of five cards sequential in value, regardless of suit (ex:  $3\clubsuit, 4\heartsuit, 5\heartsuit, 6\diamondsuit, 7\heartsuit, 8\clubsuit$ ). A *flush* is a hand consisting of five cards of the same suit, regardless of value (ex:  $10\heartsuit, 4\heartsuit, 6\heartsuit, J\heartsuit, 7\heartsuit$ ).

For the questions below, give your answer as a whole number.

How many hands constitute a *flush* in a standard 5-card draw of poker if...

- 5A** We do not include a straight flush (5 cards sequential in value AND the same suit) or a royal flush (5 cards sequential in value AND the same suit AND starting with a 10, e.g.,  $10\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit, A\heartsuit$ )

How many hands constitute a *straight* in a standard 5-card draw of poker if aces are high and...

- 5B** ...we define a straight to be inclusive of a straight-flush and royal-flush?

- 5C** ...we define a straight to exclude straight flushes and royal flushes?

