CS1800
Fall 2025
Homework 3
Assigned: October 7, 2025
Deadline: October 14, 2025 at 9pm eastern
Late Deadline: October 16, 2025 at 9pm eastern

Submission Format

CS1800 Homeworks must be neatly handwritten, on physical paper or a tablet. **Typed submissions will not be accepted.**

Submit a PDF or images to Gradescope. If you submit images, make sure they are JPEG, JPG, or PNG. (Don't submit iPhone HEIC images!) Each problem starts on a new page. Problem 1 is on a different page than Problem 2. Problems 1A and 1B can be on the same page.

Collaboration and Academic Integrity

We encourage you to work with classmates on these problems; list all collaborators on the first page of your submission. You may also consult outside sources, including generative AI; list all outside sources you referenced and how you used them on the first page of your submission.

However, **you must write all your solutions yourself, in your own words**. Do not submit anything you can't explain. Copying solutions from another person or an outside source is a violation of our academic integrity policy.

Deadline and Late Submissions

You can submit any/all homeworks up to 48 hours late with no penalty. However, **once the late deadline has passed, no submissions will be accepted**. This policy exists so you take extra time when you're especially busy, not feeling well, tending to your family, etc.; we won't make any exceptions to this policy.

Second-Chance Homework: You will have an opportunity at the end of the semester to submit one homework (HWs 1-6 only) for a new grade. If you miss both the deadline *and* the late deadline on a homework, use the Second-Chance to submit it.

For full credit

- select which pages go with which questions on Gradescope, and
- show ALL your work including intermediate steps.

Your Name

Your Collaborators (Classmates)

Outside References and How You Used Them

Problem #1 - Set Equality *(4+4+4=12 points)*

For each pair of set expressions below, determine whether the resulting sets are equal.

- **If yes...** Apply the laws of set equality to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give example elements for *A, B, C*, and the universal set *U* if necessary, that would yield a counterexample. Plug in your choices for *A, B, C* and *U* to show the sets are not the same.

For full credit, both *yes* and *no* answers should be clear, precise, and walk through your solution one small step at a time.

1A $\overline{(A \cap (A \cup \overline{B}))}$

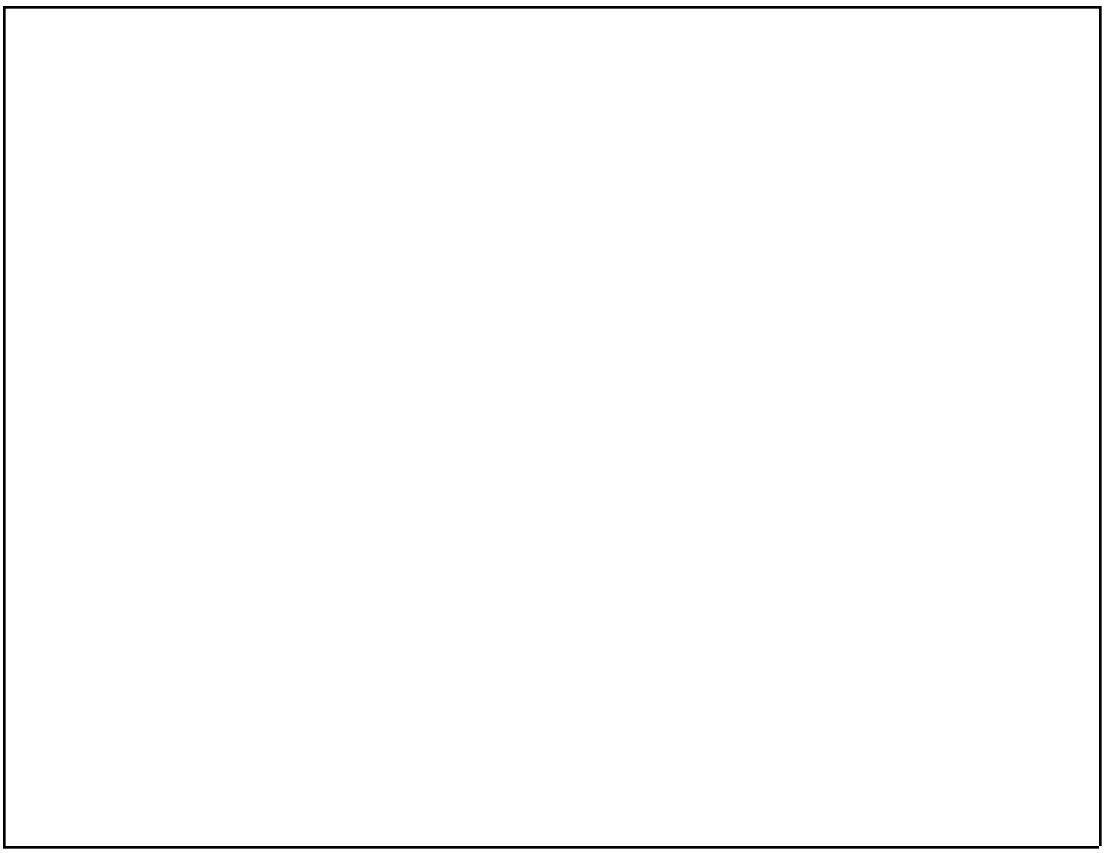
$A \cap B$

1B $\overline{(\overline{A} \cup \overline{B})} \cup (\overline{A} \cap B)$

B

1C $\overline{(A - B)}$

$B \cup \overline{(A \cup B)}$



Problem #2 - Set Notation *(2+2+2+2+2=10 points)*

Consider the subsets $A = \{1, 2, 3, 6, 9\}$ and $B = \{3, 5\}$ of the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Express each of the following sets in roster notation.

2A $\{2x + 1 \in U \mid x \in A\}$

2B $\{ x \in A \mid 2x + 1 \in U \}$

2C $\{ x \in (B \cap A) \mid 2x \in U\}$

2D $(A \cap B) \times B$

2E $P(\overline{(A \triangle B)} - \overline{(A \cup B)})$

Problem #3 - Set Expressions & Cardinality *(4+4+4=12 points)*

Dunkin' Donuts conducts a survey to see what kinds of coffee drinks Bostonians like. Here were the results:

- 40 like hot coffee with cream and sugar (aka, "regular")
- 37 like lattes
- 75 like iced coffee
- 19 like both regular and iced coffee
- 13 like both regular and lattes
- 10 like both iced coffee and lattes
- 4 like all three options
- 14 don't like of any of the options

For full credit, show all of your work for the questions below, including any arithmetic scratch work or Venn Diagrams if you drew them to help visualize the answers.

3A How many people were surveyed?
(Hint: the Principle of Inclusion/Exclusion can be useful here, and when you apply PIE to three sets instead of just two, the general formula is
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$)

3B How many people like regular coffee and nothing else?

3C How many people like exactly two types of coffee?

Problem #4 - Set Functions *(3+3+3+3=12 points)*

For any two sets *A* and *B*, define *op(A, B)* as the set $\{c \mid \forall a \in A \ \exists b \in B \ (c = a + b)\}$. In the subparts below, you are given *A* and *B*, and you are asked for the set *op(A,B)*. Give your answer in roster notation.

4A If $A = \{2, 3, 5\}$ and $B = \{1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 50, 51, 52, 53\}$, what is *op(A,B)*?

4B If $A = \{2, 3, 5\}$ and *B* is the set of even integers, what is *op(A,B)*?

Each of the collections listed below is either (1) a powerset of some set *A,* or (2) could not possibly be the powerset of anything. Your solution should: define *A* in roster notation, or declare that the collection could not be the powerset of anything.

4C {{},{*a*},{*b*},{*a,b*}}

4D {{},{C},{D},{H},{H, C}, {C, D}, {C, H}, {D, H}, {D, H}, {C, D, H}}

Problem #5 - Counting (Product Rule, Sum Rule) *(3+3+3+3=12 points)*

Prof. Patterson is making a Spotify playlist for a party. The playlist will consist of songs from his two favorite Broadway musicals: The Lion King and Wicked. He has all the tracks saved on Spotify -- 18 from Lion King and 30 from Wicked-- and wants to create playlists for his commute.

We consider the same songs played in a different order to be a different playlist. Show all of your work, including specifying separate cases where applicable. Report your final answer as a single number or as an expression such as n^k.

5A Suppose Prof. Patterson's playlist contains 3 songs and he HATES hearing the same song more than once. How many playlists could he create?

5B Suppose Prof. Patterson creates a 7-song playlist contains songs from one show or the other but not both. Repeated songs are OK. Now how many playlists are possible?

5C What if Prof. Patterson's playlist could contain 7 to 10 songs, but they're all from Lion King. Repeated songs are still OK. Now how many playlists are possible?

5D What if Prof. Patterson sticks to 7 songs in his playlist, but wants to play at least one from each show. Repeated songs are still OK. Now how many playlists are possible?