CS1800 Day 6

Admin:
- HW 2 due Friday (logic)
- HW 3 released Friday (sets)
- When will we see HW1 graded? (or any HW for that matter)
    - We will always get you HW back within 2 weeks of the due date
        - (and we'll often beat this deadline, 1.5x weeks or so)
    - You're always welcome to ask as well (re: exams)

Content:
- Sets (subsets, empty set, powerset)
- Set Builder Notation
- Set Operations (Union, Intersection, Complement, Difference)

Sets

A set is a collection of unique objects {a, b, c}

$= \{a, b, c\}$ My curly braces are not great... Sorry!

$$\{1,2,3,4\} = \{1,2,3,4,4\}$$

Poor form

$$= \{4,2,1,3\}$$

An item is in set or not, no item is in set more than once.

Example number sets you should be aware of:

Empty set

$\emptyset = \{\ \}$

set w/ no items

Integers

$\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$

$\mathbb{Z}$

Natural Numbers

$\mathbb{N} = \{0,1,2,3,\ldots\}$

Sometimes not included

Real Numbers

$\mathbb{R}$ contains

$-2, 0, 1/2, \pi, e$

Set Builder Notation: one way to express a set

$$A = \{ x \in \mathbb{N} \mid (3 \le x) \land (x \le 5) \}$$

A is THE SET OF x IN NATURAL NUMBERS SUCH THAT <SOME PREDICATE>

$x \in A$ x is item in A

$x \notin A$ x is item not in A

$0, 1, 2, 3, 4, 5, 6, 7, \ldots$

$A = \{3, 4, 5\}$

In Class Activity: Set Builder Practice

$\ldots -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 \ldots$

Express the set A by explicitly listing all items it contains

$A = \{ x \in \mathbb{Z} \mid |x| < 5 \}$

$\{ -4, -3, -2, -1, 0, 1, 2, 3, 4 \}$

Express the set B using set builder notation

B = set of all natural numbers x which have x mod 3 = 0 and x mod 7 =0 and x < 40

(++ list all of its items)

$B = \{ x \in \mathbb{N} \mid (x \text{ MOD } 3 = 0) \text{ AND } (x \text{ MOD } 7 = 0) \text{ AND } (x < 40) \}$

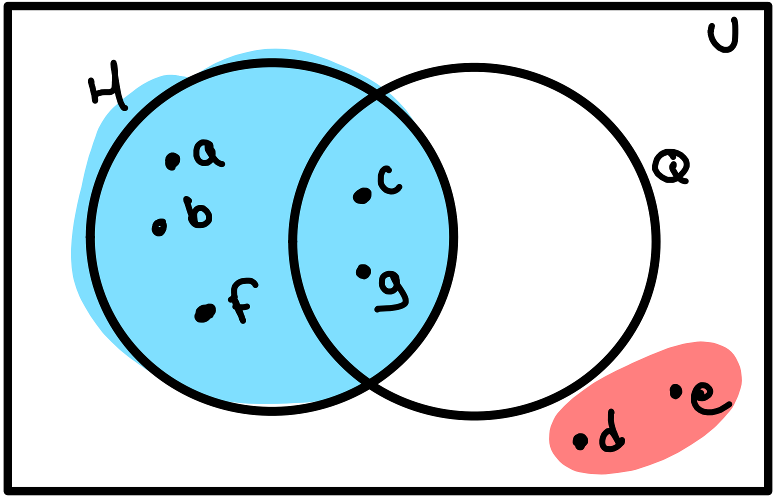
ASSUME $0 \in \mathbb{N}$

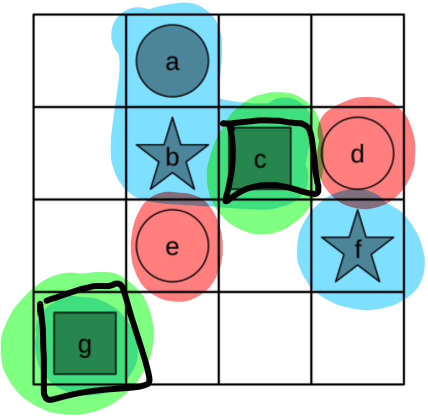
Zahlen is german for "whole number"

(where the Z for integers comes from)

Venn Diagram: a way of visually representing set membership

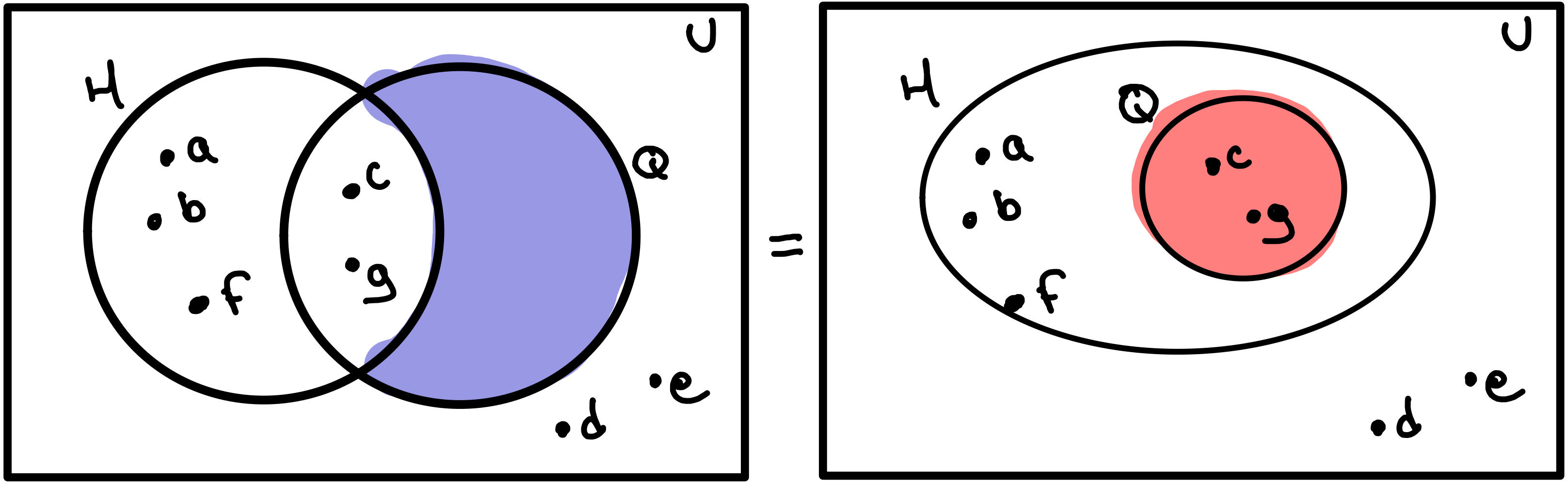
H = set of all sHaded shapes
Q = set of all sQuares
U = Universal set, contains all shapes





Venn Diagram Gotcha: Just because an area exists, doesn't mean it contains any items (may be empty)

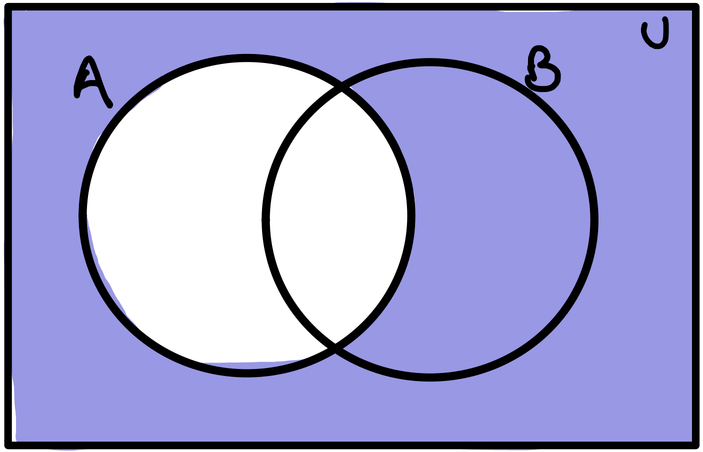
(these Venn Diagrams represent shapes from previous slide)



Generalizable representation:
This classic venn-diagram has a space for any item's set membership

This representation is valid in the special case where one set is contained in another (i.e. Q has no items not in H)

Set Operation: Complement (all the items NOT in some set)



TWO NOTATIONS FOR SAME THING

"NOT IN"

$$\overline{A} = A^c = \{ x \in U \mid x \notin A \}$$

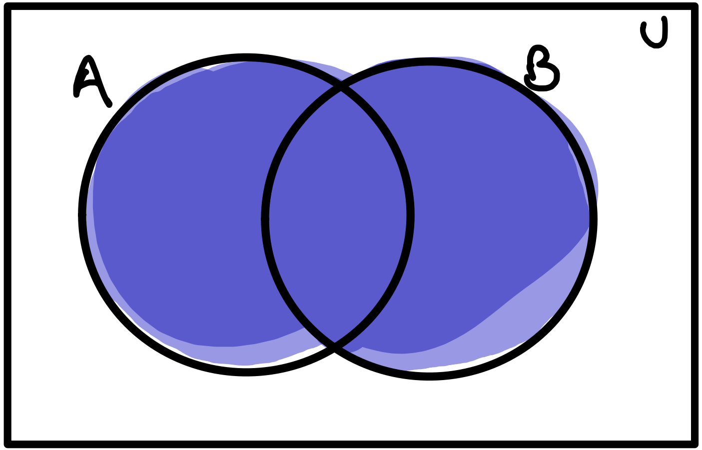
ALL x IN UNIVERSE

SUCH THAT

X IS NOT IN A

Set Operation: Union

(all the items in one set OR another)



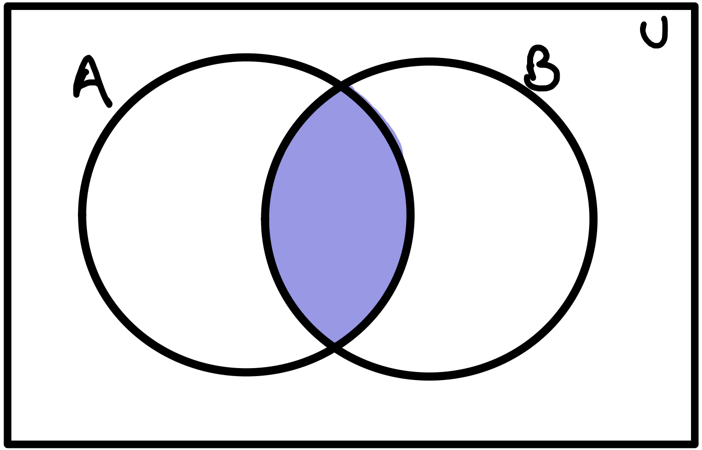
$$A \cup B = \{x \in U \mid x \in A \vee x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

X IS IN A OR X IS IN B

Set Operation: Intersection

(all the items in one set AND another)



$A \cap B = \{x \in U \mid x \in A \land x \in B\}$

ALL x IN UNIVERSE SUCH THAT

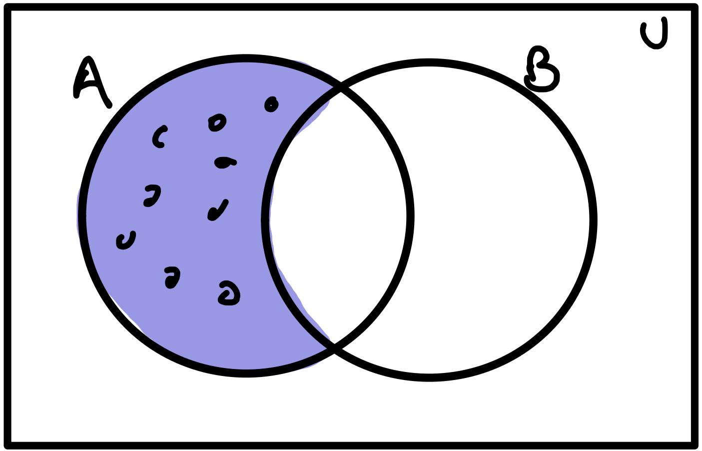
x IS IN A AND x IS IN B

Tip

$\cup$ Union "Makes sets not smaller"

$\cap$ Intersection "Makes sets not bigger"

Set Operation: Difference (All items in one set but not another)

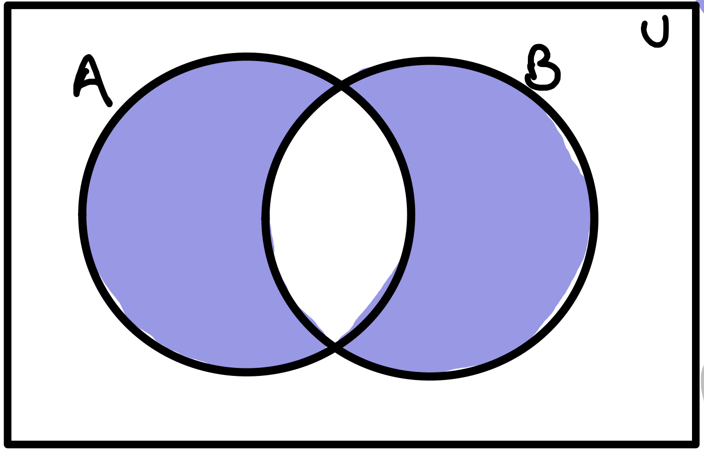


$$A - B = \{x \in U \mid (x \in A) \cap (x \notin B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN A AND X IS NOT IN B

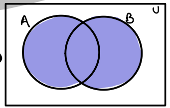
Set Operation: Symmetric Difference (All items in one set XOR another)
(All items in one set or the other, but not both)

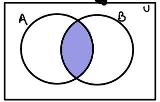


$$A \Delta B = \{x \in U \mid x \in (A \cup B) \wedge x \notin (A \cap B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN $A \cup B$ AND X NOT IN $A \cap B$

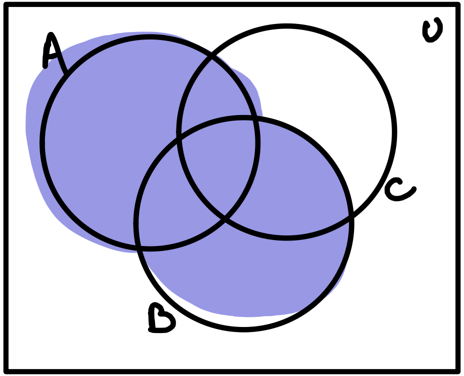




In Class Activity

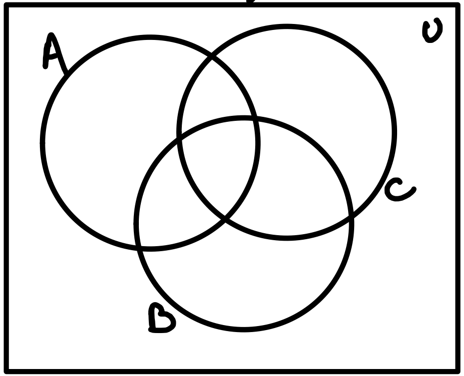
Shade the indicated areas in each venn diagram

$(A \cup B) - C$

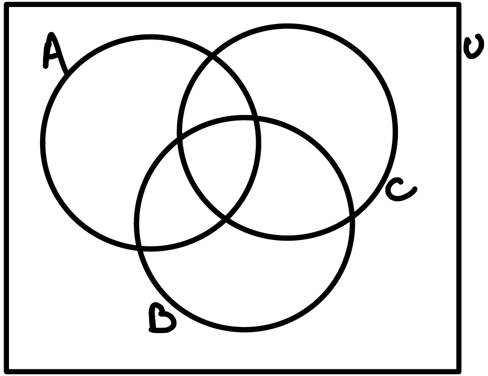


$(A \cap C) \cup B^{c}$

COMPLEMENT OPERATION
(NOT THE SET C)



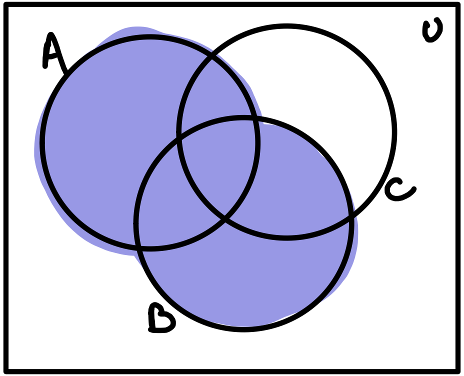
$A \triangle (B \cap C)$



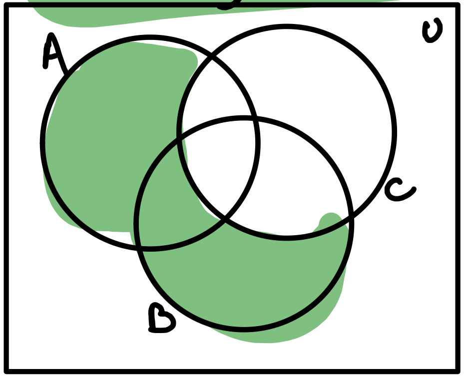
In Class Activity

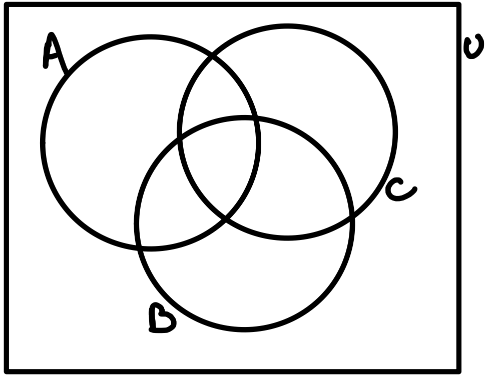
Shade the indicated areas in each venn diagram

$(A \cup B) - C$



$(A \cup B) - C$

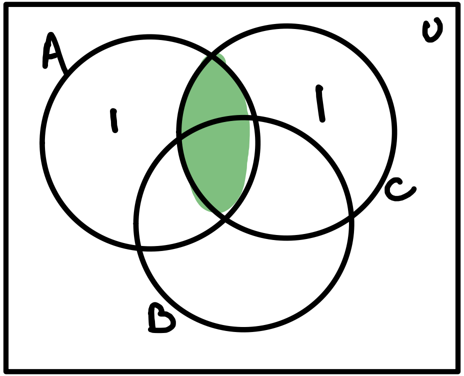




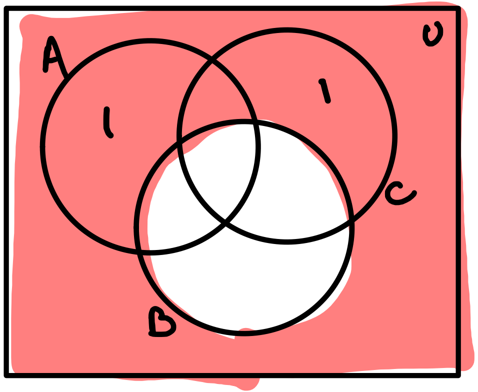
In Class Activity

Shade the indicated areas in each venn diagram

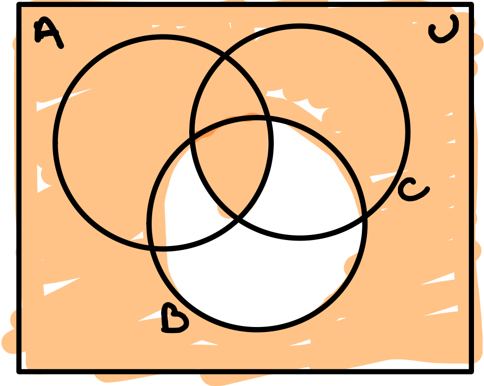
$A \cap C$



B^c



$(A \cap C) \cup B^c$

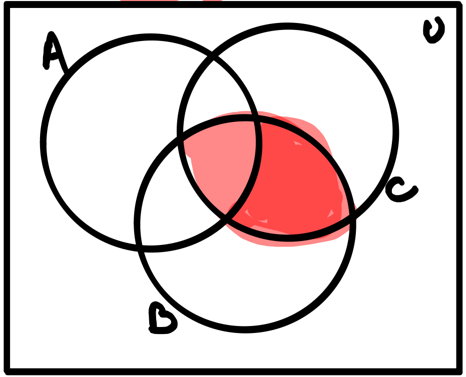


In Class Activity

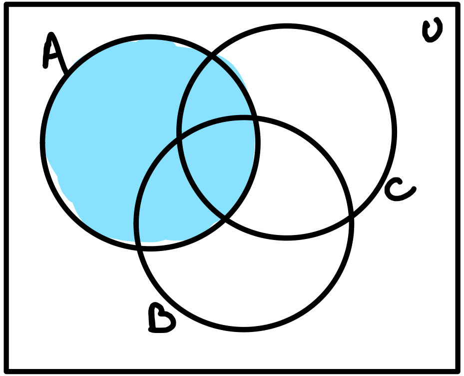
A

Shade the indicated areas in each venn diagram

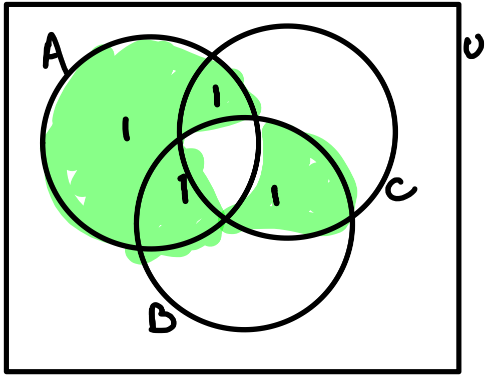
$B \cap C$



A

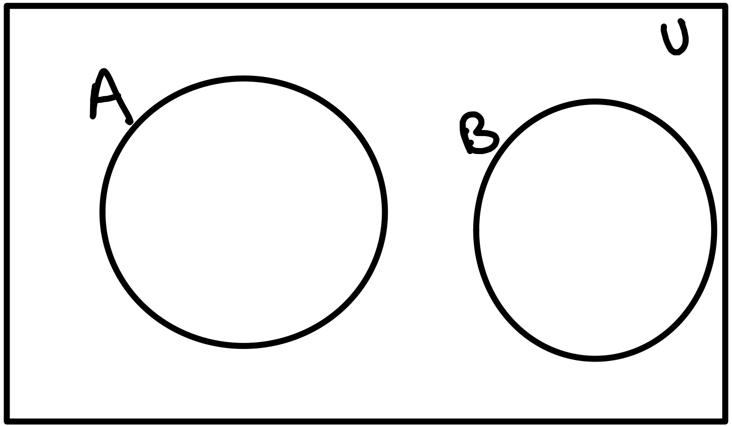


$A \,\Delta\, (B \cap C)$



Set Terminology: Disjoint Sets (two sets are disjoint if no item is in both sets)

WE SAY A, B ARE DISJOINT IF $A \cap B = \emptyset$



NO ITEM CAN BE IN BOTH A AND B

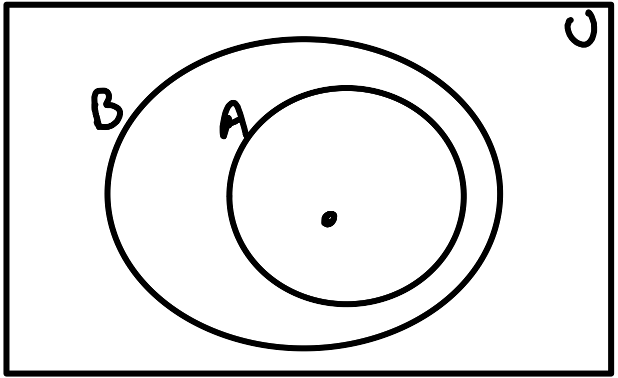
Set Terminology: subsets

A is subset of B = all items in A are in B

"IS A SUBSET OF"

$$A \subseteq B = \underline{x \in A} \ \underline{\to} \ \underline{x \in B}$$

IF X IS IN A THEN X IS IN B



WE ILLUSTRATE LIKE THIS TO SHOW $A-B=\emptyset$

(THERE IS NO ITEM IN A NOT IN B)

QUIRK: EMPTY SET IS A SUBSET OF ANY SET A

$\emptyset \subseteq A$ FOR ALL SETS A

Set Terminology: Set Equality

Given sets A, B: we say that A=B if A is a subset of B and B is a subset of A.

$A \subseteq B$

$x \in A \rightarrow x \in B$

ALL X IN A ALSO IN B

$B \subseteq A$

$x \in B \rightarrow x \in A$

ALL X IN B ALSO IN A

INTUITION: A, B HAVE SAME ITEMS

awkward at first look ... but allows for clear set equality proof approach. to show sets A = B:

- show that all items in A are in B and
- show that all items in B are in A

ALSO KIND OF ODD:

$A \subseteq B$ IS TRUE WHEN A, B ARE EQUAL

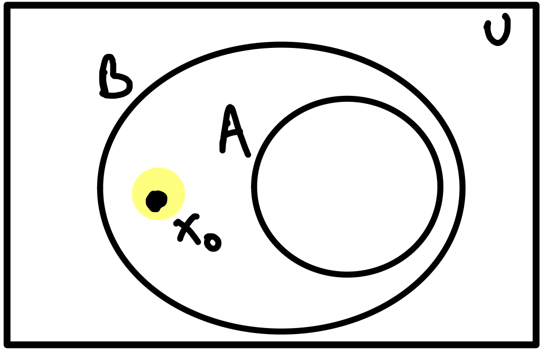
WHAT LANGUAGE CLARIFIES THAT

$A \subseteq B$ AND B IS "BIGGER" ?

Set Terminology: Proper Subset (one set is contained in another, larger, set)

$A \subset B$ = ALL ITEMS OF A ARE IN B AND B CONTAINS SOME ITEM NOT IN A = $A \subseteq B$ AND $B - A \neq \emptyset$

"A IS PROPER SUBSET OF B"



NOTICING NOTATION

$A \subseteq B$

"SET A IS A SUBSET OF B"

$A \subset B$

"SET A IS A PROPER SUBSET OF B"

$x \leq 123$

UNDERLINE "MIGHT BE EQUAL"

$x < 123$

Set Terminology: Cardinality (the number of items in a set)

$A = \{a, b, c, d\}$

$|A| = 4$

Set Terminology: Power Set

The power set of set A is the set of all sets which can be made from items in A

$A = \{1, 2\}$

$B = \{\text{☺}, \text{☹}\}$

$P(A) = \{\{2, 1\}, \{1\}, \emptyset, \{2\}\}$

$P(B) = \{\{\text{☺☹}\}, \{\text{☺}\}, \emptyset, \{\text{☹}\}\}$

In Class Activity

A = {3, 4, 5}
B = {4, 5}
C = {5}

$B \subseteq A$

Compute each of the following

$|A|$ $=3$
$|A \cup B|$ $=3$ SAME AS $|A|$ SINCE $B \subseteq A$
$|P(C)|$
$|P(B)|$
$|P(A)|$

POWERSET OF A

$P(C) = \{\{5\}, \phi\}$

$P(B) = \{\{5\}, \phi, \{5,4\}, \{4\}\}$

$P(A) = \{\{5\}, \phi, \{5,4\}, \{4\}, \{5,3\}, \{3\}, \{5,4,3\}, \{4,3\}\}$

$P(D) = 2^{|D|}$

In Class Activity

A = {3, 4, 5}

B = {4, 5}

C = {5}

Compute each of the following

|A|

|A ∪ B|

|P(C)|

|P(B)|

|P(A)|

Powerset of A

A Heavy Hint

Items in P(A)

3	4	5	
F	F	F	$\emptyset$
F	F	T	{5}
F	T	F	{4}
F	T	T	{4,5}
T	F	F	{3}
T	F	T	{3,5}
T	T	F	{3,4}
T	T	T	{3,4,5}

4,5 are in