CS 1800 Day 4

Admin:
- hw1 due Friday
- hw2 released Friday
- please please read the HW instructions (group members, tagging pages etc)
  (warning: announcement coming reminding everyone midweek too ... apologies for spamming you)

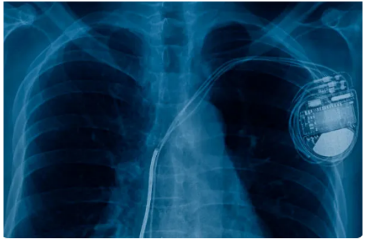
Content:
- logic statements & predicates
- truth tables
- logic operators (AND, NOT, OR)

(just an intro to these topics, we'll do more next lesson too)
- existential / universal quantifier
- conditionals

When should machine:
- give a soda
- return change

When should sunroof:
- open
- close

When should pacemaker:
- send pulse to muscle
  to pump blood?
- shock to restart heart



Logic gives us an unambiguous language to describe behavior

(spoken languages, like english, can be ambiguous)

STATEMENTS

Statement - a sentence which is either true or false

Which of the following are statements?

1. Today is Sept 17

2. "This big wooden horse definitely doesn't have greek soldiers inside"
    - Greeks who just put soldiers in that horse

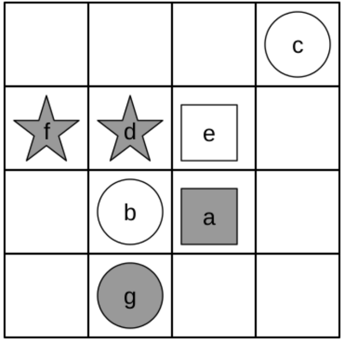
3. What is your favorite color?

4. There is intelligent life on mars

PREDICATES

Predicate - a statement about one or more variables (i.e. mad libs)

Tarski World:



circle(x) = True if shape x is a circle, False otherwise

CIRCLE(e) = FALSE

CIRCLE(c) = TRUE

next_to(x, y) = True if shapes x, y are next to each other (diagonals count too), False otherwise

NEXT_TO(b,a) = TRUE

NEXT_TO(f,g) = FALSE

0, 1   TRUE, FALSE

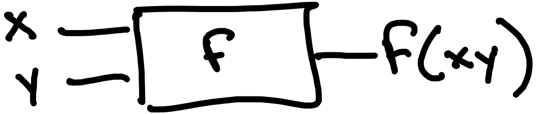
CONVENTION: BITS AND BOOLEANS

0 = FALSE   1 = TRUE

preference: use T and F when talking about Booleans: True and False

we'll use 0 and 1 when discussing circuits (e.g. electronics)

Truth Tables



We'll often describe a function of one or more inputs (e.g. vending machine operation)

A Truth Table specifies an output associated with every possible combinations of inputs

X	Y	f(xy)
F	F	
F	T	
T	F	
T	T	

X	Y	Z	g(xyz)
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

A helpful convention:

Order the rows of the truth table as if you're counting in binary
- False becomes 0
- True becomes 1

X	Y
F	F
F	T
T	F
T	T

X	Y	BINARY VALUE
0	0	0
0	1	1
1	0	2
1	1	3

not necesary for credit in this class, but still nice because:
- systematic way to avoid skipping row by accident
- consistent standard allows for easy comparison between us all

LOGICAL OPERATOR: NOT (NEGATION)

CHANGES TRUTH VALUE

"NOT X"

X	$\neg X$
F	T
T	F

Ex

X = "IT'S RAINING"

$\neg X$ = "IT'S NOT RAINING"

LOGICAL OPERATOR: AND (CONJUNCTIVE)

ONLY TRUE WHEN **ALL** INPUTS ARE TRUE

"X AND Y"

X	Y	$X \wedge Y$
F	F	F
F	T	F
T	F	F
T	T	T

D = DRIVER LICENSE PRESENTED

P = PASSPORT PRESENTED

$D \wedge P$ = DRIVER'S LICENSE AND PASSPORT PRESENTED

LOGICAL OPERATOR: OR (DISJUNCTIVE OPERATOR)

ONLY TRUE WHEN **ANY** INPUT IS TRUE

"X OR Y"

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

D = DRIVER'S LICENSE PRESENTED

P = PASSPORT PRESENTED

$D \vee P$ = DRIVER'S LICENSE OR PASSPORT PRESENTED

EXCLUSIVE OR: XOR

ONLY TRUE WHEN **EXACTLY ONE** INPUT IS TRUE

"WILL YOU HAVE GREENS OR SOUP?"

G = YOU HAVE GREENS

S = YOU HAVE SOUP

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

NOTICE DIFFERENCE FROM OR

$G \oplus S$ = "EITHER SOUP OR GREENS, NOT BOTH"

INCLUSIVE OR

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

EXCLUSIVE OR

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

"Convention": Most of the time when folk speak "or" they intend the inclusive or

but not all the time ... good luck! ;)

CONVENTION

$$\neg A \vee B \;=\; (\neg A) \vee B$$

Assume the negation operation applies to statement immediately to its right.

If the negation applies to multiple statements, use parenthases as below:

$$\neg (A \vee B)$$

Truth tables allow us to build complex expressions in bite-size steps.

GOAL: TRUTH TABLE FOR $(X \vee Y) \wedge \neg Z$

X	Y	Z	$X \vee Y$	$\neg Z$	$(X \vee Y) \wedge \neg Z$
F	F	F	F	T	F
F	F	T	F	F	F
F	T	F	T	T	T
F	T	T	T	F	F
T	F	F	T	T	T
T	F	T	T	F	F
T	T	F	T	T	T
T	T	T	T	F	F

In Class Assignment:

Build a truth table for each of the two expressions below. Results for both might feel familiar, thats ok :)

$\neg(A \vee B)$

A	B	$A \vee B$	$\neg(A \vee B)$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

$\neg A \wedge \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

$\vee$

OR

$\wedge$

AND

LOGICAL (BOOLEAN) EQUIVILENCE

Two statements are logically equivalent if their truth table columns are identical.

Statements which are logically equivalent:

- always have the same truth value (True or False)
- may be substituted for each other
  - like one does in our familiar algebra (e.g. x = 3 into 10 = x + y)

Example: logically equivalent statements:

"This shape has exactly four sides of equal length at right angles to each other"

"This shape is a square"

Previous slide demonstrates logical equivilence of:

$$\neg(A \vee B) = \neg A \wedge \neg B \quad \text{(DE MORGAN'S LAW)}$$

There are other laws too:

- helpful to simplify an expression

- we'll study these alongside set algebra & circuits, which are related topics, more to come later …

Associative Laws

$(P \lor Q) \lor R = P \lor (Q \lor R)$

$(P \land Q) \land R = P \land (Q \land R)$

Double Negation

$\neg \neg P = P$

DeMorgan's Laws

$\neg(P \lor Q) = \neg P \land \neg Q$

$\neg(P \land Q) = \neg P \lor \neg Q$

Distributive Laws

$P \land (Q \lor R) = (P \land Q) \lor (P \land R)$

$P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$

Absorption Laws

$P \land (P \lor Q) = P$

$P \lor (P \land Q) = P$

Complement Laws

$P \lor \neg P = T$

$P \land \neg P = F$

Idempotent Laws

$P \lor P = P$

$P \land P = P$

Identity

$\text{False} \lor P = P$

$\text{True} \land P = P$

Domination:

$\text{True} \lor P = \text{True}$

$\text{False} \land P = \text{False}$

Conditional Statement: (AKA Implication)

"IF x THEN y"

JOIN X	FUN Y	$X \to Y$
F	F	T
F	T	T
T	F	F
T	T	T

x = you join tutoring group
y = you have fun doing math

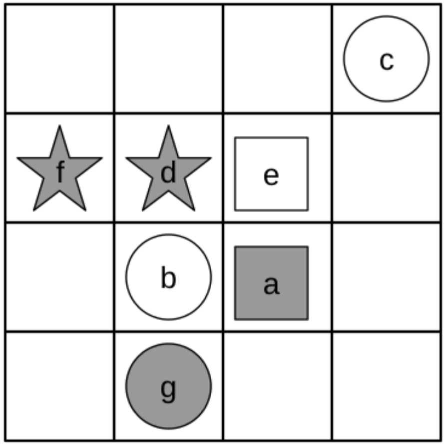
$x \to y$ = if you join a tutoring group,
then you'll have fun doing math

you haven't joined a tutoring group,
statement true by convention (will motivate later)

counter-example: you joined a tutoring group but
didn't have fun doing math, statement is False

you joined a tutoring group and had fun doing math

LOGICAL QUANTIFIER: UNIVERSAL (AKA FOR ALL)



$\forall$ x shade(x)

For every object x, x is shaded

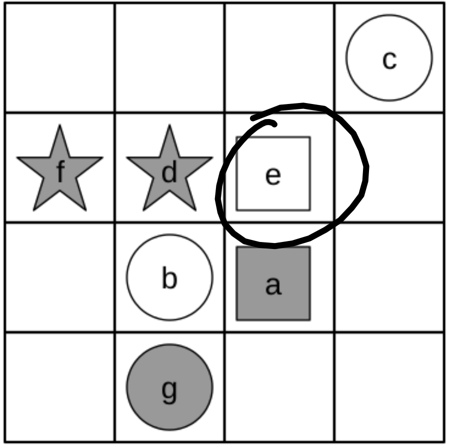
This statement is False for Tarksi world at left,
consider that c is not shaded

Another way of saying the same thing as $\forall$ x shade(x) is:
shade(a) $\wedge$ shade(b) $\wedge$ shade(c) $\wedge$ shade(d) $\wedge$ ...

in spoken language:
"For any" can be exchanged with: for all, for each, for every, in all cases

QUICK PRACTICE

IS FOLLOWING STATEMENT TRUE?



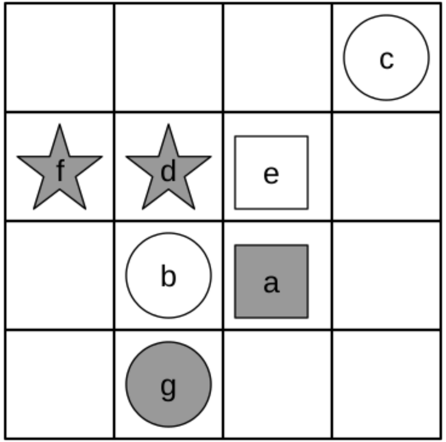
$$\forall x\,(\mathrm{STAR}(x) \rightarrow \mathrm{SHADE}(x))$$

For all shapes x, if x is a star, then x is shaded.

$$(\mathrm{STAR}(e) \rightarrow \mathrm{SHADE}(e)) \land (\mathrm{STAR}(f) \rightarrow \mathrm{SHADE}(F)) \land$$

$$g \qquad g$$

LOGICAL QUANTIFIER: EXISTENTIAL (AKA "THERE EXISTS")



∃ x shade(x)

there exists object x where x is shaded

This statement is True for Tarksi world at left, consider that object d is shaded

Another way of saying the same thing as ∃ x shade(x) is:
shade(a) ∨ shade(b) ∨ shade(c) ∨ shade(d) ∨ ...

in spoken language:
"there exists" can be replaced with: there is, at least one, it is possible to find, some

USEFUL TIP

$\forall$

FOR ALL

UPSIDE DOWN A

$\exists$

THERE EXISTS

BACKWARDS E

In Class Activity:

Using logical operators (AND, OR, NOT) quantifiers (for all, there exists) and conditionals (if-then), translate each statement below:

LET x BE STUDENT

Logic to english:

SHOES(x) = STUDENT WEARING SHOES

$\exists x \; \text{SHOES}(x) \vee \text{DANCE}(x)$

There exists at least one student who is a great dancer or wears shoes.

There exists a student where student is wearing shoes or student is a good dancer

$\forall x \; \text{DANCE}(x) \rightarrow \neg \text{SHOES}(x)$

DANCE(x) = STUDENT IS A GREAT DANCER

for all students that are great dancers then they're not wearing shoes

For all students, if the student is a good dancer then they're not wearing shoes.

English to logic (define your own statements & predicates as needed)

- You shall not pass! - Gandalf

$\forall x \neg PASS(x)$ x CAR

$\neg PASS(MONSTER)$

- I've got a wallet, keys and a phone in my pocket.

$W \wedge K \wedge P$

W =
K =
P =

- I never leave the house without my blue shoes or a hat
- "There's no place like home" - Dorothy in Wizard of Oz
- "Everybody loves you when you're 6 feet underground" -John Lennon

$POCKET(W) \wedge POCKET(K) \wedge POCKET(P)$

- I never leave the house without my blue shoes or a hat

X IS A MOMENT LEAVE HOUSE

$\forall x \ SHOE(x) \lor HAT(x)$

$\neg(\neg SHOE(x) \land \neg HAT(x))$

SHOE(x) = LEAVE w/ BLUE SHOE

HAT(x) = LEAVE w/ HAT

- "There's no place like home" - Dorothy in Wizard of Oz

$\neg \exists x \ HOME(x)$

X IS A PLACE

$\forall x \neg HOME(x)$

- "Everybody loves you when you're 6 feet underground" -John Lennon

$\forall x \ 6FEET \rightarrow LOVE(x)$