1 Dijjkstra's Shortest Path



Using Dijkstra's algorithm, find the shortest path from node A to G. Please provide a table which shows the path weight and predecessor from A to every node, labelling the visited node at each step.

Solution										
iteration	node visited	А	В	С	D	Е	F	G		
0	А	start:0	A: 2	A: 7	none	none	none	none		
1	В	start:0	A: 2	B: 3	B: 3	B: 4	none	none		
2	\mathbf{C}	start:0	A: 2	B: 3	B: 3	B: 4	C: 11	none		
3	D	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	none		
4	\mathbf{E}	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	E: 10		
5	\mathbf{F}	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	E: 10		

The path with min weight is: G \leftarrow E \leftarrow B \leftarrow A

The solution immediately above is sufficient for full credit. A step-by-step explanation is given below:

- Start algorithm: visit start node A
- add path $A \to B$ with cost 2 (no previous path to B)
- add path $A \rightarrow C$ with cost 7 (no previous path to C)

iteration	node visited	А	В	С	D	Е	F	G
0	А	start:0	A: 2	A: 7	none	none	none	none

- Continue algorithm: visit node with min cost among unvisited: B has cost 2
- update to path $A \to B \to C$ with cost 3 (previous path $A \to C$ had cost 7)
- add path $A \to B \to D$ with cost 3 (no previous path to D)
- add path $A \to B \to E$ with cost 4 (no previous path to E)

iteration	node visited	А	В	С	D	Е	F	G
1	В	start:0	A: 2	B: 3	B: 3	B: 4	none	none

• Continue algorithm: visit node with min cost among unvisited: C has cost 3

• ignore path $A \to C \to D$ with cost 8 (previous path $A \to B \to D$ had cost 3)

• add path $A \to C \to F$ with cost 11 (no previous path to F)

iteration	node visited	А	В	С	D	Е	F	G
2	С	start:0	A: 2	B: 3	B: 3	B: 4	C: 11	none

- Continue algorithm: visit node with min cost among unvisited: D has cost 3
- ignore path $A \to \dots \to D \to E$ with cost 6 (previous path $A \to B \to E$ had cost 4)
- update to path $A \to ... \to D \to F$ with cost 5 (previous path $A \to C \to F$ had cost 11)

iteration	node visited	А	В	С	D	Е	F	G
3	D	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	none

- Continue algorithm: visit node with min cost among unvisited: E has cost 4
- ignore path $A \rightarrow ... \rightarrow E \rightarrow F$ with cost 10 (previous path $A \rightarrow ... \rightarrow D \rightarrow F$ had cost 5)
- add path $A \rightarrow ... \rightarrow E \rightarrow G$ with cost 10 (no previous path to G)

iteration	node visited	А	В	С	D	E	F	G
4	Е	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	E: 10

	\bullet Continue algorithm: visit node with min cost among unvisited: F has cost 5									
	• ignore path A $\rightarrow \dots \rightarrow F \rightarrow G$ with cost 13 (previous path A $\rightarrow \dots \rightarrow E \rightarrow G$ had cost 10)									
• End algorithm: next to visit is destination: G has cost 10										
	iteration	node visited	А	В	С	D	Е	F	G	
	5	F	start:0	A: 2	B: 3	B: 3	B: 4	D: 5	E: 10	