Prof. Hamlin
Day 4

Agenda

1) HW due friday - please read submission instructions
2) Review
3) Logic, predicates
4) Truth tables
5) logic operators (AND, OR, NOT)

looking ahead

Quantifiers
Conditionals (if → then)

Review

1) Negative binary
   a) sign bit
   b) two's complement

Practice:

1) -19 as 6-bit two's complement

$-2^{6-1} = -32 \longrightarrow -32 + x = -19$

$x = 13$

$13 = 6 \cdot 2 + 1$
$6 = 3 \cdot 2 + 0$
$3 = 1 \cdot 2 + 1$
$1 = 0 \cdot 2 + 1$

10110_2

Logic

Human language is very imprecise
~and~
computers don't speak it

How do we communicate ideas like
- "if I hit the power button, the screen should turn on."
- "$2.50 and pressing C3 dispenses a gingerale"
- "Cats make people happier than dogs"

Logic unambiguous - only one way to interpret it

Statement - a sentence that is either true or false
(T) (F)

e.g. "Professor Hamlin owns a cat"

Which are statements
1. Today is Tuesday - Yes
2. What is your favorite class? - No
3. Corina is better than Zeke. - Yes
4. CS1800 teaches logic. - Yes

Preclicate: a stament about one or more variables

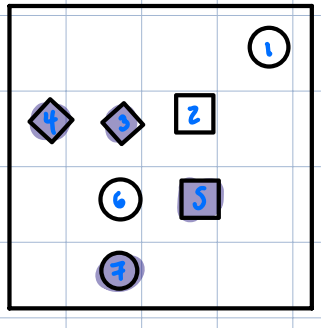
e.g. The cactus (noun) is yellow. (Mad libs)

in logic is written as

this is a statement

Yellow(x): the object x is yellow

Tarski World



Circle(x): object x is circle

Circle(1): True Circle(5): false

- Right_of(x,y): object x is in a column right of object y

Right_of(3,2): False Right_of(5,4): true

Boolean - something that is either True (T) or false (F)

e.g. a statement evaluates to a boolean

Sometimes computers will use bits to rep. booleans

0: false 1= true

Putting it together: Consider the statements

A = "Own a cat" B = "is cool"

We can think of these statements as boolean values. What are the possible combinations

Own a cat	is cool	
F	F	(Prof. Higgers)
F	T	
T	F	≠
T	T	(Me)

This is called a Truth table

Truth Table: a list of every possible input for boolean values

Often include columns that represent the output of more complex logical operations....

Logical Operators

Arithmetic operators: +, −, ÷, *

Boolean operators: AND, OR, NOT, XOR

Not - negation

Symbol: $\neg$

x = "own cat"

$\neg x$ = doesn't own cat

x	$\neg x$
F	T
T	F

AND - only true when all inputs are true (conjunction)

Symbol: $\wedge$

x = "owns cat" y = "is cool"

$x \wedge y$ = "owns cat and is cool"

x	y	$x \wedge y$ ← "x and y"
F	F	F
F	T	F
T	F	F
T	T	T

(inclusive or)

OR - only true when one or more input is true (disjunction)

Symbol: $\vee$

x = "want cookie" y = "want donut"

$x \vee y$ = "want cookie or donut"

x	y	$x \vee y$ ← "x or y"
F	F	F
F	T	T
T	F	T
T	T	T

XOR: exactly one of the inputs are true (exclusive or)

Symbol: $\oplus$

x = "own a dog" y = "are cool"

$x \oplus y$ = "either own a dog or you are cool but not both"

x	y	$x \oplus y$
F	F	F
F	T	T
T	F	T
T	T	F

false is 0, true = 1

$0 + 0 \mod 2 = 0$

$0 + 1 \mod 2 = 1$

$1 + 0 \mod 2 = 1$

$1 + 1 \mod 2 = 0$

Note: English is ambiguous - if or is used in a sentence is it exclusive or (inclusive) OR

↑ like this!

Order of operations:

Arithmetic: PEMDAS → (), exp, mult, div, add, sub

Logic: PNAXO → (), NOT, AND, XOR, OR

$\neg A \vee (B \wedge C)$ ① $(B \wedge C)$

② $\neg A$

③ $\vee$

Truth tables can be built a bit at a time for complex boolean logic statements

e.g. $(x \vee y) \wedge \neg z$

X	Y	Z	¬Z	X ∨ Y	(X∨Y) ∧ ¬Z
F	F	F	T	F	F
F	F	T	F	F	F
F	T	F	T	T	T
F	T	T	F	T	F
T	F	F	T	T	T
T	F	T	F	T	F
T	T	F	T	T	T
T	T	T	F	T	F

Exercise Build Truth tables for

1) $\neg(A \vee B)$ 2) $\neg A \wedge \neg B$

A	B	A ∨ B	¬(A∨B)
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

A	B	¬A	¬B	¬A ∧ ¬B
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

These two expressions are logically equivalent

Logical equivalence - two statements are logically equivalent if Truth tables are identical

e.g. "The shape has right angles and 4 equal connected sides"

"The shape is a square"

$x = 3$
$x + 9$
$3 + 9$

Note: can also substitute them

$$\neg(A \vee B) \wedge \neg Z = \neg A \wedge \neg B \wedge \neg Z$$

Conditional statements (implication) - if x then y
Symbol: $x \rightarrow y$

x = "own a cat" y = "are cool"
$x \rightarrow y$ = "if you own a cat then you are cool"

x	y	$x \rightarrow y$
F	F	T
F	T	T
T	F	F
T	T	T

in red case only T by convention

= don't own cat no evidence if imp is true or not

= implication is false someone owns a cat and is not cool

= implication is true cat owners are cool

Exercise: write Truth Table for

$(x \to y) \vee z$

x	y	z	$x \to y$	$(x \to y) \vee z$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	F
T	F	T	F	T
T	T	F	T	T
T	T	T	T	T

Quantifiers

What if I wanted to make a statement about all the students in the class doing math

Doing_Math(Riya) ∧ Doing_Math(Andrew) ∧ Doing_Math(Hana)...

That is a lot to write out! Easier way is quantifiers

∀ (universal, for all) (upside down A)

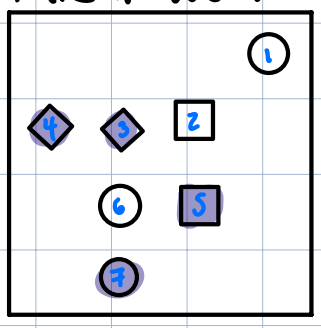
$\forall$ x in CS1800 students, doing_Math(x)

for all student x in CS1800, x is doing math

for every, for each

every statement anded together is true

Exercise
Tarski World



for all objects x, if x diamond, then x is shaded

$\forall x : diamond(x) \rightarrow shade(x)$

diamonds 3, 4
shaded

True

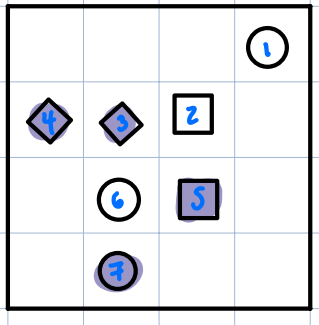
∃ (existential, there exists) (at least one) (backward E)

Name is Ariel (Hana) ∨ Name is Ariel (Riya) ∨ ....

$\exists$ x in CS1800 student, Name is Ariel (x)

There exists at least one student in CS1800 whose name is Ariel

Exercise
Tarski World



$\exists x : shaded(x) \wedge circle(x)$

There exists at least one shape that is both shaded and circle

True,

Exercise:

1) Logic to English:

x is student, shoes(x) = wearing shoes, dance(x) = can dance

a) $\exists x : shoes(x) \vee dance(x)$

There exists a student who wears shoes or can dance

b) $\forall x : dance(x) \rightarrow \neg shoes(x)$

for all students, if they can dance they are

not wearing shoes

2) English to logic (define own predicates & statements
e.g if there exists a cat is on lap then I am happy
on_lap(x) = is cat on lap happy = I am happy

$\exists x$: on_lap(x) $\rightarrow$ happy

a) I have wallet, keys, and phone
$W \wedge K \wedge P$

b) Everyone loves a hero
$\forall x$: love_hero(x)

c) If I leave home then I (have) shoes or a hat
leave_home $\rightarrow$ (shoes $\vee$ hat)

d) There exists a place like home

$\exists x$: like_home(x)