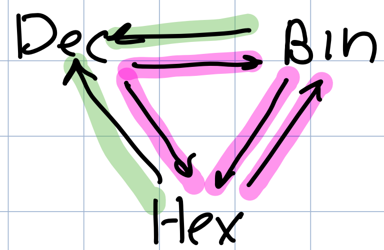
Prof. Hamlin
Day 2

Agenda

1) Review
2) 

= last class
= today

3) adding / mult in another base
3) modular arithmetic

What did we cover last time?

Binary Conversion

Practice

1) 1011_2

$1 \cdot 1000_2 + 0 \cdot 100_2 + 1 \cdot 10_2 + 1 \cdot 1_2$

$1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 1 \cdot 1$

$\boxed{11}$

2) 123_4

$1 \cdot 100_4 + 2 \cdot 10_4 + 3 \cdot 1_4$

$1 \cdot 16 + 2 \cdot 4^1 \quad 3 \cdot 1$

27

Decimal to other bases

14_{10} in base 2

Remember powers of 2		
$2^0 = 1$	=	1_2
$2^1 = 2$	=	10_2
$2^2 = 4$	=	100_2
$2^3 = 8$	=	1000_2
$2^4 = 16$	=	10000_2

14 in between (2^3 and 2^4)

How many times does 16 go in? 0 $\quad 0 \cdot 10000_2$

.... 8? 1 $\quad 1 \cdot 1000_2$

But we now need to handle only the remaining value - we have handled 8.

$14 - 8 = 6$

. . . 4? 1 $\quad 1 \cdot 100_2$ → co-efficients

Handle only remaining value $\quad 6 - 4 = 2$

. . . 2? 1 $\quad 1 \cdot 10_2$

Remaining value $\quad 2 - 2 = 0$

... 1 ? 0 $0 \cdot 1_2$

Remaining value $0 - 0 = 0$

All together...

$0 \cdot 10000_2 + 1 \cdot 1000_2 + 1 \cdot 100_2 + 1 \cdot 10_2 + 0 \cdot 1_2$

1110_2

Practice and 13_{10}

16 & 13?	$10000_2 \cdot 0$	R 13
8 & 13?	$1000_2 \cdot 1$	R 5
4 & 5?	$100_2 \cdot 1$	R 1
2 & 1?	10_2 0	R 1
1 & 1?	$1_2 \cdot 1$	R 0

1101_2

We can also think about doing this backwards if we don't have our handy table of powers

Euclid's Division

(Subtraction)
First method - start w/ largest power of two and work down

Euclid's - start w/ smallest power and get bigger

Recipe: (Okay to just memorize!)

0. Start w/ decimal value
1. Divide value w/ base get remainder + multiplier
2. Stop if multiplier is 0 otherwise continue step 1
3. Read remainders BOTTOM TO TOP

$14_{10} \rightarrow ?_2$

$14 = 7 \cdot 2 + 0$
$7 = 3 \cdot 2 + 1$
$3 = 1 \cdot 2 + 1$
$1 = 0 \cdot 2 + 1$

Stop here

1110_2

Practice: Express 23_{10} as binary using
1. Subtraction
2. Euclid's

Extra: How are these methods similar / different?

1)

16	2	23	1·16	R	7
8	2	7	0·8	R	7
4	2	7	1·4	R	3
2	2	3	1·2	R	1
1	2	1	1·1	R	0

10111_2

2)

$2\overline{)23}$ = 11 R1

$23 = 11 \cdot 2 + 1$
$11 = 5 \cdot 2 + 1$
$5 = 2 \cdot 2 + 1$
$2 = 1 \cdot 2 + 0$
$1 = 0 \cdot 2 + 1$

10111_2

Note Euclid works for other bases as well!

$23_{10} \rightarrow$ base-6

$23 = 3 \cdot 6 + 5$ ↑

$3 = 0 \cdot 6 + 3$

$\boxed{35_6}$

Math in other bases

Addition

$$\begin{array}{r} {}^{1}\,1\ 2\ 3_{10} \\ +\ 2\ 8\ 1_{10} \\ \hline 4\ 0\ 4 \end{array}$$

$$\begin{array}{r} {}^{1}\,3\ C\ 4_{16} \\ +\ 1\ 5\ 2_{16} \\ \hline 5\ 1\ 6_{16} \end{array}$$

What is C

$A = 10$

$B = 11$

$C = 12$

$12_{10} + 5_{10} = 17_{10}$

$= 16_{10} + 1_{10}$

↓

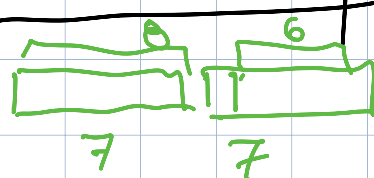
11_{16}

Multiplication

$$\begin{array}{r} {}^{1}\,1\ 2\ 3_{10} \\ \times\ \ 4\ 1_{10} \\ \hline 1\ 2\ 3 \\ +\ 4\ 9\ 2\ 0 \\ \hline 5\ 0\ 4\ 3 \end{array}$$

$$\begin{array}{r} {}^{1}\,1\ 7\ 2_{8} \\ \times\ 2\ 1_{8} \\ \hline 1\ 7\ 2_{8} \\ 3\ 6\ 4\ 0_{8} \\ \hline 4\ 0\ 3\ 2_{8} \end{array}$$

Remember all the digits are already base 8



→ If stuck try a similar decimal problem

Practice

1)
$$\begin{array}{r} {}^{1}\ {}^{1}\ \ \\ 1\,4\,7_8 \\ +\ 4\,4_8 \\ \hline 2\,1\,3_8 \end{array}$$

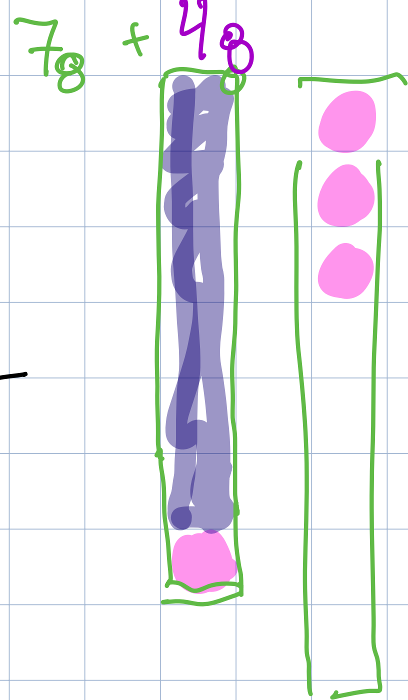
2)
$$\begin{array}{r} {}^{1}\ \ \ \\ 3\,2_4 \\ \times\ 2\,2_4 \\ \hline 1\,1\,3\,0 \\ 1\,3\,0\,0 \\ \hline 2\,0\,3\,0_4 \end{array}$$

$7+4 \rightarrow 11_{10} \rightarrow 8+3 \rightarrow 13_8$

$9_{10} \rightarrow 8+1 \rightarrow 11_8$

$4_{10} \rightarrow 10_4$

$3 \cdot 2 + 1 = 7_{10} \rightarrow 4+3 \Rightarrow 13_4$



Modular Arithmetic

Who here has ever done modular arithmetic?

You all have 

Currently 9 PM if I start a 4 hr movie marathon what time will I go to bed?

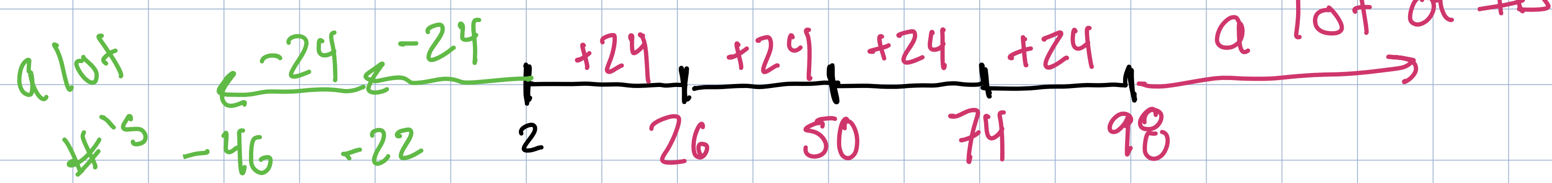
9 + 4 = 1 AM (not military time)

9 PM + 2 hrs = 11 PM
9 PM + 26 hrs = 11 PM
9 PM + 50 hrs = 11 PM

These times are equal on clock if differ by 24

Modulo: an intuition

What are the numbers which add 2 hrs to 9?



↑ this represents all these #s

x mod n

A definition: modulo is the remainder when you divide the number by the mod

98 mod 24 = 2

$$24 \overline{)98} = 4 \text{ R } 2$$

Practice

1) 15 mod 4 = 3 $\qquad 4 \overline{)15} = 3 \text{ R } \boxed{3}$

2) 14 mod 7 = 0 $\qquad 7 \overline{)14} = 2 \text{ R } \boxed{0}$

Negative Numbers? Same idea w/ one more step

$-28 \bmod 5$

$28 \bmod 5$

$5 \overline{)28}$ = 5 R 3

But the remainder is actually negative!

$-3 \bmod 5$

Just add mod value

$-3 + 5 = \boxed{2}$