Prof. Hamlin
Day 4

Agenda

1) HW due friday - please read submission instructions
2) Review
3) Logic, predicates
4) Truth tables
5) logic operators (AND, OR, NOT)

looking ahead

Quantifiers
Conditionals (if → then)

Review

1) negative binary #
   a) signed bit
   b) two's complement

Practice:

1) -19 as 6-bit two's complement

$-2^{6-1} = -32$ → $-32 + x = -19$

← $x = 13$

$13 = 6 \cdot 2 + 1$
$6 = 3 \cdot 2 + 0$
$3 = 1 \cdot 2 + 1$
$1 = 0 \cdot 2 + 1$

1	0	1	1	0	1 $_2$

Logic

Human language is very imprecise
~and~
computers don't speak it

How do we communicate ideas like
- ... "if I hit the power button, the screen should turn on."
- ... "$2.50 and pressing C3 dispenses a gingerale"
- ... "Cats make people happier than dogs"

Statement - a sentence that is either true or false
(T) (F)

e.g. "Professor Hamlin owns a cat"

Which are statements
1. Today is Tuesday — Yes, True
2. What is your favorite class? — Not
3. Corina is better than Zeke. — Yes, True
4. CS1800 teaches logic. — Yes

Preclicate : a stament about one or more variables

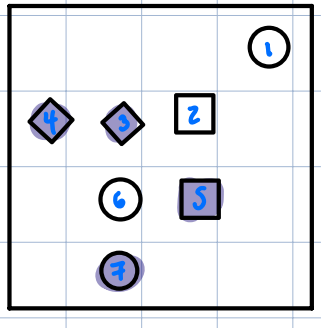
e.g. The Dog (noun) is yellow. (Mad libs)

in logic is written as

isYellow(x) : the object x is yellow

isYellow(Dog)

Tarski World



Circle(x) object x is a circle

Circle(1): True Circle(5): False

- Right_of(x,y): object x is in a column right of y

Rgnt_of(3,2): False Right_of(5,4): True

Boolean - something that is either True (T) or false (F)

e.g. a statement evaluates to a boolean

Sometimes comp will use bits to represent boolean value

0 = false 1 = true

Putting it together: Consider the statements

A = "Own a cat" B = "is cool"

We can think of these statements as boolean values. What are the possible combinations

Own a cat	is cool	
F	F	(Professor Hagger)
F	T	
T	F	
T	T	(Me)

This is called a Truth table

Truth Table: a list of every possible input for boolean values

Often include columns that represent the output of more complex logical operations....

Logical Operators

Arithmetic operators: +, −, %, *

Boolean operators: AND, OR, NOT, XOR

Not - negation

Symbol: ¬

x = "own cat"

$\neg x$ = "does not own cat"

x	$\neg x$
F	T
T	F

AND - only true when all inputs are true (conjunction)

Symbol: ∧

x = "owns cat" y = "is cool"

$x \wedge y$ = "owns cat and is cool"

"x and y"

x	y	$x \wedge y$
F	F	F
F	T	F
T	F	F
T	T	T

(inclusive or)

OR - only true when one or more input is true (disjunction)

Symbol: ∨

x = "want cookie" y = "want donut"

$x \vee y$ = "want cookie or donut"

"x or y"

x	y	$x \vee y$
F	F	F
F	T	T
T	F	T
T	T	T

XOR: exactly one of the inputs are true (exclusive or)

Symbol: $\oplus$

x = "own a dog" y = "are cool"

$x \oplus y$ = "either own a dog or are cool but not both"

~ x xor y T = 1 F = 0

x	y	$x \oplus y$	
F	F	F	→ 0 + 0 mod 2 = 0
F	T	T	→ 0 + 1 mod 2 = 1
T	F	T	1 + 0 mod 2 = 1
T	T	F	→ 1 + 1 mod 2 = 0

Note: English is ambiguous - if or is used in a sentence is it exclusive or (inclusive) OR

↳ like this!

Order of operations:

Arithmetic: PEMDAS → (), exp, [mult, div] [add, sub]

$(2 \cdot 3 + 1)^2 = 49$

Logic: PNAXO → (), NOT, AND, XOR, OR

$\neg A \vee (B \wedge C)$

① $B \wedge C$

② $\neg A$

③ $\vee$ operator

$A \wedge B \wedge \neg C$

Truth tables can be built a bit at a time for complex boolean logic statements

e.g. $(x \vee y) \wedge \neg z$

X	Y	Z	$\neg z$	$x \vee y$	$(x \vee y) \wedge \neg z$
F	F	F	T	F	F
F	F	T	F	F	F
F	T	F	T	T	T
F	T	T	F	T	F
T	F	F	T	T	T
T	F	T	F	T	F
T	T	F	T	T	T
T	T	T	F	T	F

Exercise Build Truth tables for

1) $\neg(A \vee B)$ 2) $\neg A \wedge \neg B$

A	B	$A \vee B$	$\neg(A \vee B)$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

two statements are logically equivalent

Logical equivalence - two statements are logically equivalent if Truth tables are identical

e.g. "The shape has right angles and 4 equal connected sides"

"The shape is a square"

A $X=3$
B $X+9$
$3+9$

Note: can also substitute them

$$\neg(A \vee B) \wedge \neg Z = \neg A \wedge \neg B \wedge \neg Z$$

Conditional statements (implication) - if X then Y

Symbol: $X \rightarrow Y$

X implies Y

X = "own a cat" Y = "are cool"

$X \rightarrow Y$ = "if you own a cat then you are cool"

X	Y	$X \rightarrow Y$
F	F	T
F	T	T
T	F	F
T	T	T

= don't own a cat so can be cool or not

= implication is false own a cat but not cool

= implication is true own cat and are cool

if X is false nothing to prove imp. false so defaults to true

Exercise: write Truth Table for $(x \to y) \vee z$

x	y	z	$x \to y$	$(x \to y) \vee z$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	F
T	F	T	F	T
T	T	F	T	T
T	T	T	T	T

Quantifiers

What if I wanted to make a statement about all the students in the class doing math

Doing_Math(Riya) ∧ Doing_Math(Andrew) ∧ Doing_Math(Hana)...

That is a lot to write out! Easier way is quantifiers

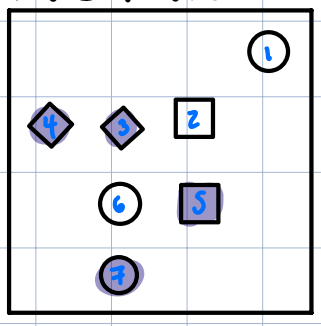
∀ (universal, for all) (upside down A)

$\forall x$ in CS00 students, doing_Math(x)

for every student x in CS00, x is doing math.

for all / for each mean same thing

Exercise
Tarski World



for all x shapes, if shape x is diamond then it is shaded

$\forall x : diamond(x) \rightarrow shade(x)$

Diamonds: 3, 4

Shaded: Yes

Yes! statement is true

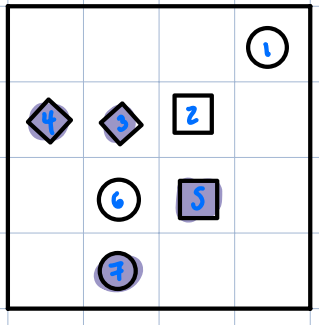
∃ (existential, there exists) (at least one) (backward E)

NameIsAriel(Riya) ∨ NameIsAriel(Hana) ∨ NameIs Ariel(Andrew)...

there exists at least one student in CS800 whose name is Ariel

$\exists x$ in CS800 student, NameIsAriel(x)

Exercise
Tarski World



there exists a shape that is both shaded and circle

$\exists x : shaded(x) \wedge circle(x)$

shaded shapes: 4, 3, 5, 7

circles: 6, 7, 1

True!

Exercise:

1) Logic to English:

x is student, shoes(x) = wearing shoes, dance(x) = can dance

a) $\exists x : shoes(x) \vee dance(x)$

There exists a student who wears shoes or can dance

b) $\forall x : dance(x) \rightarrow \neg shoes(x)$

for all student, if they can dance then they

don't wear shoes

2) English to logic (define own predicates & statements
e.g if there exists a cat is on lap then I am happy
on_lap(x) = T/F cat on lap happy = am I happy

$\exists x: onlap(x) \rightarrow happy$

a) I have wallet, keys and phone
A = wallet B = having keys C = having phone

$A \wedge B \wedge C$

b) Everyone loves a hero

$\forall x: loves\_hero(x)$

c) If I leave home then I have shoes or a hat

$leave\_home \rightarrow (shoes \vee hat)$

$\uparrow \oplus$

d) There exists a place like home

$\exists x : like\_home(x)$