Professor Hamlin
Day 5

Agenda
1) Admin
2) Review
3) Extended Conditionals
   - contrapositive, inverse, converse
   - double implication
4) Extended Quantifiers
   - negation
   - combining

Review

Vocab: Statements, Predicate, Boolean, Truth tables

Logical ops: → , AND, OR, NOT, XOR

Quantifiers: $\forall$ → for all, bunch of 'and' statements

$\exists$ → there exists, one or more statements true

Exercise: 1) Construct TT for following expression

$$(x \vee \neg y) \wedge x$$

x	y	$x \vee \neg y$	$(x \vee \neg y) \wedge x$
F	F	T	F
F	T	F	F
T	F	T	T
T	T	T	T

2) Convert the following logic to english

a) $\forall x: cat(x) \rightarrow zoomies(x)$

For all pets x, if x is a cat then x has zoomies.

b) $\exists x: student(x) \wedge (inCS1800(x) \vee sad(x))$

There exists a student who is in 1800 or is sad.

Extended Conditionals

Remember TT for $x \to y$

x	y	$x \to y$
F	F	T
F	T	T
T	F	F
T	T	F

← true by convention

← only false when we have counter example

Consider the following:

G = life gives you lemons

M = you make lemonade

Which of these sentences seem equivalent??

$\to$, $\neg$

1) if life gives you lemons, then you make lemonade

$G \to M$

2) If you are not making lemonade, then life hasn't given you lemons

$\neg M \to \neg G$

3) If you make lemonade, then life has given you lemons

$M \to G$

4) If you haven't been given lemons, then you aren't making lemonade

$\neg G \to \neg M$

Exercise: convert the statements above into formal logic

Its hard to think about <u>logical equivalence</u> so let's compare TT

G	$\neg G$	M	$\neg M$	Given Make $G \to M$	Not make Not given $\neg M \to \neg G$	Make Given $M \to G$	Not Given Not make $\neg G \to \neg M$
F	T	F	T	T	T	T	T
F	T	T	F	T	T	F	F
T	F	F	T	F	F	T	T
T	F	T	F	T	T	T	T

original = contra-positive   inverse = converse

Consider: $x \to y$

contrapositive: $\neg y \to \neg x$, if not y then not x
equivalent to original statement

converse: $y \to x$, if y then x
→ not equivalent to original statement → can make lemonade w/o being given lemons
→ equivalent to inverse

inverse: $\neg x \to \neg y$, if not x, then not y
→ equivalent to converse

original = contrapositive   converse = inverse
original ≠ converse/inverse

Negating implications

We have all these fancy terms but what about just $\neg(G \to M)$?

Its not actually any of them! A negation is when all T become F and F become T so.

x	y	$x \to y$	$\neg(x \to y)$
F	F	T	F
F	T	T	F
T	F	F	T
T	T	T	F

What is an equivalent statement?

Exercise: Try and discover an equivalent statement (this will take trial and error)

x	y	$\neg x \vee y$	$x \wedge \neg y$
F	F	T	F
F	T	T	F
T	F	F	T
T	T	T	F

$\neg, \wedge$

$\neg(\neg x \vee y)$

$\neg(\neg x) \wedge \neg y$

$x \wedge \neg y$

$$\boxed{\neg(x \to y) = x \wedge \neg y}$$

Double implication (Bi-conditional)

$$x \leftrightarrow y$$

if x then y AND if y then x

$$(x \rightarrow y) \wedge (y \rightarrow x)$$

This means x can only happen if y does and visa versa.

English shorthand: "if and only if" or "iff"

What is the Truth table?

x	y	$x \rightarrow y$	$y \rightarrow x$	$x \leftrightarrow y$ $(x \rightarrow y) \wedge (y \rightarrow x)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

Exercise: Convert english to logic, create statements and predicates as needed

1) I'll wear a rainjacket if and only if it's raining

Rainjacket $\leftrightarrow$ Rain

2) You can be cool if and only if you own a cat

Cool $\leftrightarrow$ cat_ownership

Extended Quantifiers

Negating Quantifiers:

"All students in the class love cats"

What is the opposite of this statement in English?

"No one in the class loves cats"

However quantilers are more explicit if

$\forall x: \text{loves\_cat}(x)$

is false, it means there is at least one student who dislikes cats, not that everyone dislikes cats.

love_cat(Hana) $\wedge$ love_cat(Andrew) $\wedge$ love_cat(Matt) ...
T T F

So we can say there exists at least one student who doesnot like cats...

$\exists x: \neg \text{like\_cats}(x)$

$\neg(\forall x: P(x)) \Rightarrow$
$\exists x: \neg(P(x))$

$$\boxed{\neg(\forall x: P(x)) \longleftrightarrow \exists x: \neg P(x)}$$

Consider

"there exists a student with a birthday today"

$$\exists x : \text{Birthday}(x)$$

If its false no one has a birthday today

~or~

for every student, their birthday isn't today

$$\forall x : \neg \text{Birthday}(x)$$

$$\boxed{\neg(\exists x : P(x)) \longleftrightarrow \forall x : \neg P(x)}$$

Exercise: Consider the sentence & logic, negate it, and English

"for all lemons: if I receive it then I make lemonade"

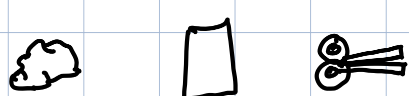
$$\neg(\forall \ell : G(\ell) \to M(\ell)) \Rightarrow \exists \ell : \neg(G(\ell) \to M(\ell))$$

$$\Rightarrow \exists \ell : G(\ell) \wedge \neg M(\ell)$$

There exist a lemon: then when given it,
I don't make lemonade

Combining Quantifiers:

Everyone in class, lets play Rock, Paper, Scissors!



But.. who wins?

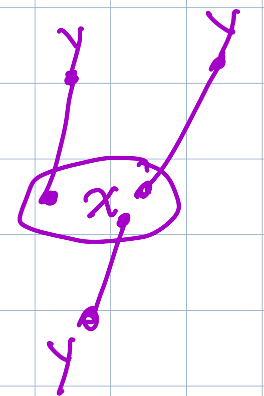
Find another student who you beat
everyone should find one!

$Win(x,y)$ = x beats y at RPS   True!

For every student x, there exists another student y where x beats y

$$\forall x : \exists y : Win(x,y)$$

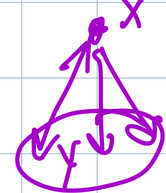
$\boxed{x \text{ gets to choose its own } y}$



Alright is there exist a student, who for all other students they have won against them?

No, But we can turn this statement to logic

$$\exists x \ \forall y \ Win(x,y)$$



The same x has to work for every y

Exercise: Express as logic.

1) Everyone has somebody who can make them smile.

$$\forall x \ \exists y : smile(x,y)$$

2) There is someone, against everyone else, ran a faster race.

$$\exists x \ \forall y : faster(x,y)$$ ← x faster than y

Exercise: negate

1) $\neg(\forall x \exists y : \text{smile}(x,y)) \Rightarrow \exists x \, \neg(\exists y : \text{smile}(x,y))$

$\Rightarrow \exists x \, \forall y : \neg \text{smile}(x,y)$

2) $\neg(\exists x \, \forall y : \text{faster}(x,y))$ y is slower than x

$\Rightarrow \forall x \, \neg(\forall y : \text{faster}(x,y))$

$\Rightarrow \forall x \, \exists y : \neg \text{faster}(x,y)$