CS1800 Day 2 (to 3:15PM)

Admin:
- HW1 released Friday (not available yet)
- bring a pen/pencil/paper and work with laptop open from notes (they're on website)
- any questions?

Content:
Converting Between Bases:
  - subtract-largest-power-of-base method (intuitive)
  - euclid's division method (easier ... we'll see later they're the same)

Operating (adding & subtracting) in other bases

Modular Arithmetic:

Division on integers: Floor Division & Remainder

We can't (currently) represent non-whole numbers.

How does division work if we restrict ourselves to whole numbers?
(i.e. integers are all whole numbers {..., -3, -2, -1, 0, 1, 2, 3...})

7 // 3 = 2

FLOOR DIVISION

Floor division works just like normal division, but we always round down to nearest whole number

(in example above, 7/3= 2.3333... so 7 // 3 = 2)

Division on integers: Floor Division & Remainder

Sometimes we're interested in the remainder (motivation to come shortly)

$7 \;//\; 3 = 2$

FLOOR DIVISION

$7 = 3 \cdot 2 + 1$

"7 divided by 3 is

2 with a remainder of 1"

In Class Activity: Division on Integers:

Compute each of the integer divisions below by computing:

- floor division
- remainder

clearly label which is which (super helpful as we build on this idea shortly)

7 divided by 3	$7 // 3 = 2$	$7 = 3 \cdot 2 + 1$
25 divided by 2	$25 // 2 = 12$	$25 = 2 \cdot 12 + 1$
100 divided by 7	$100 // 7 = 14$	$100 = 7 \cdot 14 + 2$

What is the largest and smallest remainder produced by dividing any value by 5?
1 divided by 5, 2 divided by 5, 3 divided by 5, ...

What is the largest and smallest remainder produced by dividing any value by 5?

smallest remainder is 0, happens when a number is divisible by 5
largest remainder is 5 (excluding 5, all values which are less than 5, but not 5)

1 divided by 5, 2 divided by 5, 3 divided by 5, ...

$0 // 5 = 0$

$1 // 5 = 0$

$2 // 5 = 0$

$3 // 5 = 0$

$4 // 5 = 0$

$5 // 5 = 1$

$6 // 5 = 1$

$7 // 5 = 1$

$0 = 0 \cdot 5 + 0$

$1 = 0 \cdot 5 + 1$

$2 = 0 \cdot 5 + 2$

$3 = 0 \cdot 5 + 3$

$4 = 0 \cdot 5 + 4$

$5 = 1 \cdot 5 + 0$

$6 = 1 \cdot 5 + 1$

$7 = 1 \cdot 5 + 2$

Modular Arithmetic: Motivation via wall-clock time

If the time now is 4 PM:

- what time is it in 1 hour?
- what time is it in 25 = 1 + 24 * 1 hours?
- what time is it in 49 = 1 + 24 * 2 hours?
- what time is it in 73 = 1 + 24 * 3 hours?
- what time is it in 1 + 24 * n hours (for a whole number n)?

1//24 = 0	$1 = 0 \cdot 24 + 1$	1 mod 24 = 1
25//24 = 1	$25 = 1 \cdot 24 + 1$	25 mod 24 = 1
73//24 = 3	$73 = 3 \cdot 24 + 1$	73 mod 24 = 1

Punchline:
When counting time, values are equivilent if they differ by a factor of 24 (e.g. 24, 48, 72 etc)

Notice:
All these values (..., -47, -23, 1, 25, 49, 73, ...) all have remainder 1 when floor dividing by 24

Modulo operator:
X mod 24 = remainder when floor dividing X by 24

0 Mod 2 = 0
1 Mod 2 = 1
2 Mod 2 = 0
3 Mod 2 = 1
4
5
6

In Class Activity (modulo cool-down, number representation warm-up):

- solve for x:

$$11 \bmod 4 = x$$

$11 // 4 = 2$

$11 = 2 \cdot 4 + 3$

$11 \bmod 4 = 3$

- Find 4 integers X which all have X mod 3 = 2

Convert each of the following back to decimal (base-10):

$(1011)_2$

2^4 2^2	2^1	2^0	
1	0	1	1

$8 + 2 + 1 = 11$

$(123)_4$

4^2	4^1	4^0
1	2	3

$1 \cdot 16 + 2 \cdot 4 + 3 \cdot 1$

$16 + 8 + 3 = 27$

$X \bmod 3 = 2$

$$\left(\text{ANY NUMBER DIVISIBLE BY } 3\right) + 2$$

$$\left(2 + 3n \text{ FOR } n = 0, 1, 2, \ldots \right)$$

$$(\ldots 2, 5, 8, 11, 14, \ldots)$$

$1 \,//\, 3 = 0$

$2 \,//\, 3 = 0$

$3 \,//\, 3 = 1$

$4 \,//\, 3 = 1$

$5 \,//\, 3 = 1$

$6 \,//\, 3 = 2$

$7 \,//\, 3 = 2$

$1 = 0 \cdot 3 + 1$

$n=0 \quad 2 = 0 \cdot 3 + 2$

$3 = 1 \cdot 3 + 0$

$4 = 1 \cdot 3 + 1$

$n=1 \quad 5 = 1 \cdot 3 + 2$

$6 = 2 \cdot 3 + 0$

$7 = 2 \cdot 3 + 1$

$n=2 \quad 8 = 2 \cdot 3 + 2$

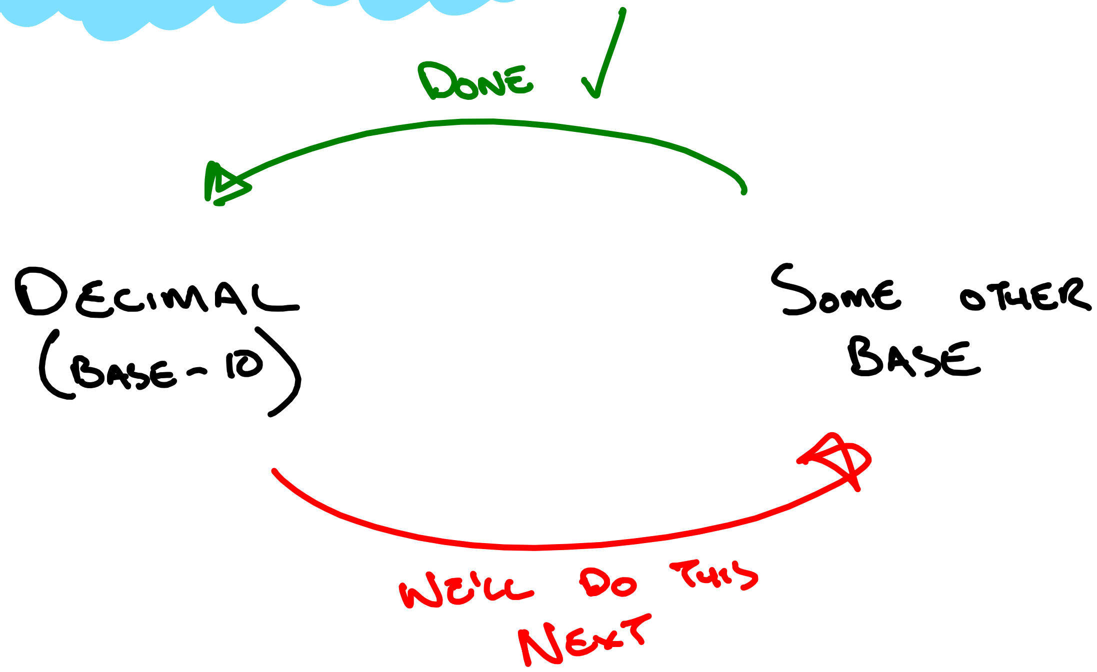
$-7 \text{ MOD } 4$

$-7 = -2 \cdot 4 + 1$

$-7 // 4 = -2$

$-7 / 4 = -1.75$

Converting Between Bases



Decimal to Another Base:

Subtract largest power of base

$B=2$

$2^0=1$

$2^1=2$

$2^2=4$

$2^3=8$

$2^4=16$

$\vdots$

Solve for x

$$14=(x)_2$$

$$14=8+6$$
$$=8+4+2$$

8	4	2	1
1	1	1	0

$$(1110)_2$$

DECIMAL TO ANOTHER BASE: EUCLID'S DIVISION METHOD

SOLVE FOR X

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

Decimal to Another Base: Euclid's Division Method

Solve for x

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

STOP HERE

$$(1110)_2$$

1. Given decimal value is first value

2. Divide value by base w/ whole numbers (use a remainder)

3 Set new value as base-multiplier

4. Repeat from step 2 until new value is 0 then stop (don't write another line)

5. Glue together all remainders (last-to-first) to produce answer

In Class Activity

32	16	8	4	2	1
	1	0	1	1	1

$(10111)_2$

Express 23 as a binary value using:

- subtract-largest-power-of-base
- Euclid's division method

(++) How are these methods similar? How are they different? How might you demonstrate that Euclid's divisoin method gives the correct answer?

$$\begin{aligned} 23 &= 16+7 \\ &= 16+4+3 \\ &= 16+4+2+1 \end{aligned}$$

$$\begin{aligned} 23 &= 11\cdot 2+1 \\ 11 &= 5\cdot 2+1 \\ 5 &= 2\cdot 2+1 \\ 2 &= 1\cdot 2+0 \\ 1 &= 0\cdot 2+1 \end{aligned}$$

$(10111)_2$

Operating (adding & multiplying) in another base

(works just like decimal, though it might feels funny at first)

Operating in other bases: addition

Perform each of the following addition operations:

0 1 2 3 4 5 6 7 8 9 A B C D E F

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

$123 + 281$

$$\begin{array}{r} \overset{1}{1}23 \\ +\ 281 \\ \hline 404 \end{array}$$

$(3C4)_{16} + (152)_{16}$

$$\begin{array}{r} \overset{1}{3}C4 \\ +\ 152 \\ \hline 516 \end{array}$$

$(516)_{16}$

$5 + 12 = 17$

$17 = 16 + 1 = (11)_{16}$

$1 + 3 + 1 = 5$

16^2	16^1	16^0

$A + 5$

$10 + 5 = F$

$17 = 16 + 1$

$A = 10$

$B = 11$

$C = 12$

$D = 13$

$E = 14$

$F = 15$

16^0	16^1	16^2
1	16	256

Operating in other bases: multiplication

Perform each of the following multiplication operations:

$123 \cdot 41$

$$\begin{array}{r} 123 \\ 41 \\ \hline 123 \\ +\ 4920 \\ \hline 5043 \end{array}$$

$(172)_8 \cdot (21)_8$

$$\begin{array}{r} 172 \\ 21 \\ \hline 172 \\ +\ 3640 \\ \hline 4032 \end{array}$$

0 1 2 3 4 5 6 7

$8^1 \ 8^0$

1	3

$2 \cdot 2 = 4$

$2 \cdot 7 = 14 = 8 + 6$

$= (16)_8$

$7 + 4 = 11 = 8 + 3$

$(13)_8$

$1 + 1 + 6 = 8 + 0 = (10)_8$

Operating in other bases (tips):

- use scratch work on the side (in decimal, to be comfortable)

- don't use base-10 values in original problem (convert to given base!)

If you get stuck, make up and write out a similar decimal example, it will prime your brain to make the same moves in the strange, alien base

In Class Activity

Perform each of the following operations in the given base:

$(147)_8 + (44)_8$

$(32)_4 \cdot (22)_4$