Agenda

Professor Hamlin
Day 5

1) Admin
2) Review
3) Extended Conditionals
   - contrapositive, inverse, converse
   - double implication
4) Extended Quantifiers
   - negation
   - combining

Review

Vocab: Statements, Predicate, Boolean, Truth table

Logical ops: AND, OR, NOT, XOR, $\rightarrow$

Quantifiers: $\exists$, $\forall$ $\rightarrow$ bunch of 'and', all true

$\exists$: $x \vee y \vee z \ldots$, one or more true

Exercise: 1) Construct TT for following expression

$$(x \vee \neg y) \wedge x$$

x	y	$\neg y$	$x \vee \neg y$	$(x \vee \neg y) \wedge x$
F	F	T	T	F
F	T	F	F	F
T	F	T	T	T
T	T	F	T	T

2) Convert the following logic to english

a) $\forall x: cat(x) \rightarrow zoomies(x)$

for all pets x, if x is a cat then x has zoomies

b) $\exists x: \neg student(x) \wedge (inCS1800(x) \vee sad(x))$

there exists a student who is in CS800 or sad.

Extended Conditionals

Remember TT for $x \to y$

x	y	$x \to y$
F	F	T
F	T	T
T	F	F
T	T	T

← True by convention

← only false when we have counter example

Consider the following:

G = life gives you lemons
M = you make lemonade

$\sim$
$\neg$

Which of these sentences seem equivalent??

$\to$, $\neg$

1) if life gives you lemons, then you make lemonade

$G \to M$

2) if you are not making lemonade, then life hasn't given you lemons

$\neg M \to \neg G$

3) if you make lemonade, then life has given you lemons

$M \to G$

4) if you haven't been given lemons, then you aren't making lemonade

$\neg G \to \neg M$

Exercise: convert the statements above into formal logic

Its hard to think about <u>logical equivalence</u> so let's compare TT

G	$\neg G$	M	$\neg M$	Given Make $G \to M$	Not make Not given $\neg M \to \neg G$	Make Given $M \to G$	Not Given Not make $\neg G \to \neg M$
F	T	F	T	T	T	T	T
F	T	T	F	T	T	F	F
T	F	F	T	F	F	T	T
T	F	T	F	T	T	T	T
					contra-positive	converse	inverse

Consider: $x \to y$

contrapositive: $\neg y \to \neg x$, if not y then not x
equivalent to original statement

converse: $y \to x$, if y then x
→ not equivalent to original statement →
can make lemonade w/o being given lemons
→ equivalent to inverse

inverse: $\neg x \to \neg y$, if not x, then not y
→ equivalent to converse

original = contrapositive   converse = inverse

original ≠ converse/inverse

Negating implications

We have all these fancy terms but what about just $\neg(G \to M)$?

Its not actually any of them! A negation is when all T become F and F become T so.

x	y	$x \to y$	$\neg(x \to y)$
F	F	T	F
F	T	T	F
T	F	F	T
T	T	T	F

What is an equivalent statement?

Exercise: Try and discover an equivalent statement (this will take trial and error)

hint to start with

x	y	$\neg x \vee y$	$x \wedge \neg y$	$\neg(x \to y)$
F	F	T	F	F
F	T	T	F	F
T	F	F	T	T
T	T	T	F	F

$\neg(\neg x \vee y)$

$\neg(\neg x) \wedge \neg y$

$x \wedge \neg y$

$$\boxed{\neg(x \to y) = x \wedge \neg y}$$

Double implication (Bi-conditional)

$$x \leftrightarrow y$$

if x then y AND if y then x

$$(x \rightarrow y) \wedge (y \rightarrow x)$$

This means x can only happen if y does and visa versa.

English shorthand: "if and only if" or "iff"

What is the Truth table?

x	y	$x \rightarrow y$	$y \rightarrow x$	$(x \rightarrow y) \wedge (y \rightarrow x)$ $x \leftrightarrow y$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

$x \equiv y$ is also $x \leftrightarrow y$

Exercise: Convert english to logic, create statements and predicates as needed

1) I'll wear a rainjacket if and only if it's raining

Rainjacket $\leftrightarrow$ Raining

2) You can be cool if and only if you own a cat

Cool $\leftrightarrow$ cat

Extended Quantifiers

Negating Quantifiers:

"All students in the class love cats"

What is the opposite of this statement in English?

"No one in the class loves cats"

However quantiers are more explicit if

$$\forall x: \text{loves\_cat}(x)$$

is false, it means there is at least one student who dislikes cats, not that everyone dislikes cats.

love_cat(Hana) ∧ love_cat(Andrew) ∧ love_cat(Matt) ...

T T F :(

So we can say there exists at least one student who doesnot like cats...

$$\exists x: \neg \text{like\_cats}(x)$$

$$\boxed{\neg(\forall x: P(x)) \longleftrightarrow \exists x: \neg P(x)}$$

Consider

"there exists a student with a birthday today"

$$\exists x : \text{Birthday}(x)$$

True :) HB Tim!

If its false no one has a birthday today

~or~

for every student, their birthday isn't today

$$\forall x : \neg \text{Birthday}(x)$$

$$\boxed{\neg(\exists x : P(x)) \longleftrightarrow \forall x : \neg P(x)}$$

Exercise: Consider the sentence & logic, negate it, and English

"for all lemons: if I receive it then I make lemonade"

$$\neg(\forall \ell : G(\ell) \to M(\ell))$$

$x \to y$

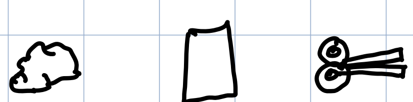
$$\exists \ell : \neg(\overset{x}{G(\ell)} \to \overset{y}{M(\ell)})$$

$$\exists \ell : G(\ell) \wedge \neg M(\ell)$$

There exists a lemon, that when given it, I don't make lemonade.

Combining Quantifiers:

Everyone in class, lets play Rock, Paper, Scissors!



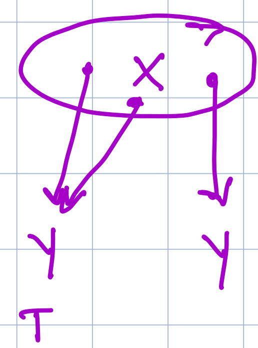
But.. who wins? Everyone.

Find another student who you beat
everyone should find one!

$$\mathrm{Win}(x,y) = x \text{ beats } y \text{ at RPS}$$

For every student x, there exists another student y
where x beats y

$$\forall x : \exists y : \mathrm{Win}(x,y)$$

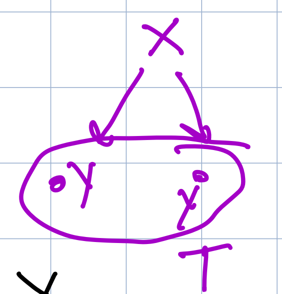


$\boxed{x \text{ gets to choose its own } y}$

Alright is there exist a student, who for all other students
they have won against them?

No true, at least think about logic

$$\exists x \ \forall y \ \mathrm{Win}(x,y)$$



The same x has to work for every y

Exercise: Express as logic.

1) Everyone has somebody who can make them smile.

$$\forall x \ \exists y : \mathrm{smiles}(x,y)$$

2) There is someone, against everyone else, ran a faster race.

$$\exists x \ \forall y : \mathrm{faster}(x,y)$$

Exercise: are the following T/F, if false find counter example

1) $\exists x \forall y$: $x+y$ is even
there exists x works for all y, false
$y=7$ $x=1$ but $y=8$ $x \neq 1$

2) $\forall x \exists y$: $x+y$ is even: true
for all x choose y
$x=19$ $y=1$
$x=20$ $y=0$

Exercise: Negate

1. $\neg(\forall x \exists y: smile(x,y))$
$\exists x \neg(\exists y: smile(x,y))$
$\exists x \forall y: \neg smile(x,y)$

2. $\neg(\exists x \forall y: faster(x,y))$
$\forall x \neg(\forall y: faster(x,y))$
$\forall x \exists y: \neg faster(x,y)$