1) Admin Exam means funny deadline | Professor Hamlin | Day 9

- includes up to day 10
- Sol'n released for HW 4 on sunday

good news: more time to study
bad news: only may use 1 late day on HW 4

2) Review
3) Permutations
4) Counting stragegies
   - Partition
   - complement
   - simplification

Review:

Pigeonhole principle $\exists$ pile w/ at least $\lceil N/k \rceil$ items

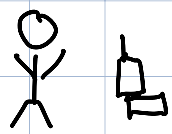
Counting: product rule (*and*) - cartesian product
sum rule (*or*)
principle inclusion/exclusion

Exercise: Can wear pants/shirt ~or~ dress
(4 pants, 3 shirts, 2 dresses) how many outfits?

$|\text{pant}| * |\text{shirts}| = 4 \cdot 3 = 12$

$12 + 2 = \boxed{14}$

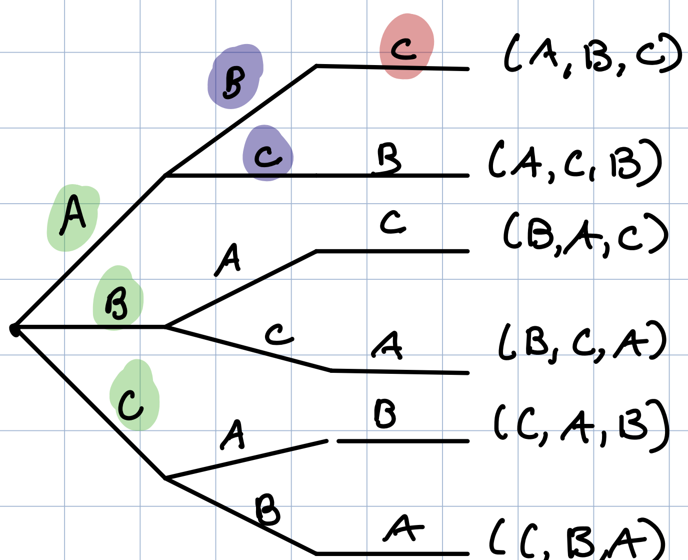
Permutations (order matters)



Vaccum salesman, visiting

Atlanta Boston Chicago

How many ways can the traveling salesman visit the three cities?



Also can think...

3 choices for first city

AND

2 choices for second city

AND

1 choice for third city

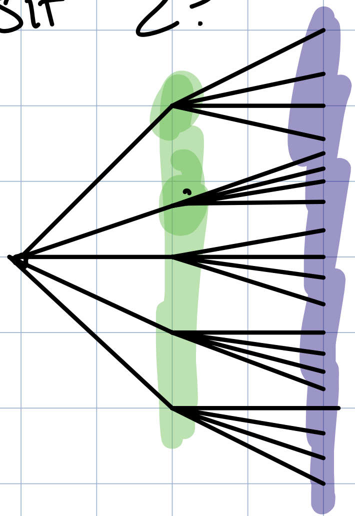
| First choice | * | Second choice | * | third choice |

3 * 2 * 1 = $\boxed{6}$

Also known as $\boxed{3!}$ (3 factorial)

> **Factorial**: $n! = n*(n-1)*(n-2)*... 3*2*1$
>
> By convention $0! = 1$

What if we have 5 cities and only visit 2?



5 choices for 1st city
AND
4 choices for 2nd city

$$5 \cdot 4 = 20$$

can also think of it as

$$5 \cdot 4 = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \frac{5!}{3!}$$

Formally: a permutation is a way of ordering k objects, from total of n

"n permute k"

$(n-k)! \neq n! - k!$

$$P(n,k) = \frac{n!}{(n-k)!}$$

$n \geq k$

$\overset{k}{3}$ of $\overset{n}{3}$ cities

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1}$$

$\boxed{3!}$ $\boxed{6}$

$\overset{k}{2}$ of $\overset{n}{5}$ cities

$$P(5,2) = \frac{5!}{(5-2)!} = \boxed{\frac{5!}{3!}}$$

$\boxed{20}$

Exercise 1) How many ways are there to order 5 people in a line for a photo?

$n=5 \quad k=5$

$$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0} = \boxed{5!}$$

2) How many ways of ordering 6 out of 20 people for photo? $n=20 \quad k=6$

$$P(20,6) = \frac{20!}{(20-6)!} = \boxed{\frac{20!}{14!}}$$

$20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15$

Bonus: How big of factorial do you need to 'break' your calculator?

$10! \approx 3 \times 10^6$

$20! \approx 2 \times 10^{18}$

Reference #'s

- 10^{17} seconds since big bang
- 10^{80} atoms in universe
- 10^{100} googol

On Hw/Exams - please leave as factorial

e.g $\frac{5!}{2!}$

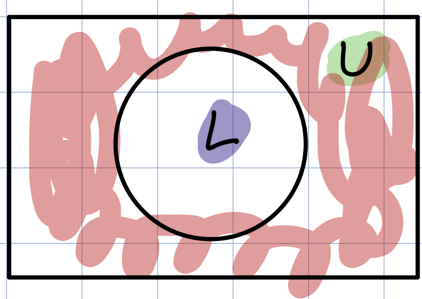
Counting stragetles:

For complex counting problems – how can we approach them?

There are a few common stragetles.

Count-by-Complement

How many ways to order 5 people s.t. A not last?



U = All orderings of 5 people

L = set of orderings where A is last

idea: if we can calculate (U) we can just subtract all items we aren't interested in (L)

$$|U-L| = |U| - |L|$$

Only works if disjoint!

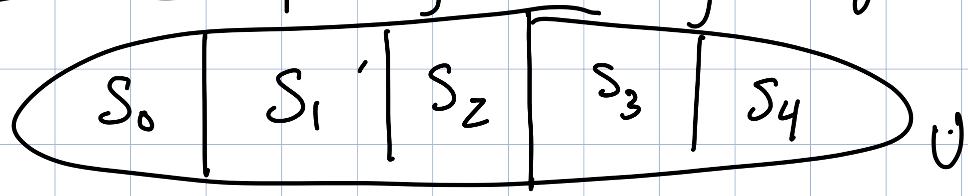
|U| = order all 5 people
P(5,5)
5!

L = A is last
P(4,4)

_ _ _ _ A
n=4
k=4

$\boxed{5! - 4!} = 96$

Partition splitting something larger into subsets



→ no overlap between subsets e.g. $S_0 \cap S_1 = \emptyset$

→ $S_0 \cup S_1 \cup S_2 \cup S_3 \cup S_4 = U$ (all elements included in subsets)

Careful: → all elements are included
→ no element is included in more than one subset

Count-by-partition

How many ways to order 5 people s.t. A not last?

A is first or A is second or A is third or A is 4th

A _ _ _ _ $\quad n=4, k=4 \quad P(4,4) = 4!$

~or~ $\quad +$

_ A _ _ _ $\quad P(4,4) = 4!$

~or~ $\quad +$

_ _ A _ _ $\quad P(4,4) = 4!$

~or~ $\quad +$

_ _ _ A _ $\quad P(4,4) = 4!$

$$\boxed{4 \cdot 4!} = 96$$

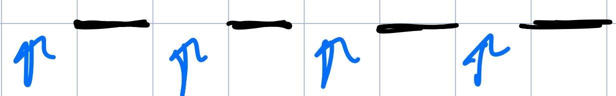
Split choices into disjoint sets e.g.

$$|A \text{ in first}| + |A \text{ in second}| + \dots + |A \text{ in } 4^{th}|$$

Common error is to have overlap between sets - be careful!

Count-by-simplification (extension of partition)

How many ways to order people5 s.t. A not last?



$\boxed{4 \cdot 4!}$

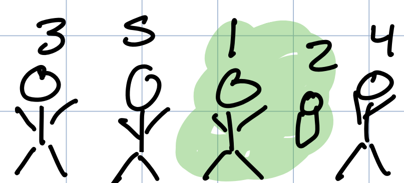
P(4,4) ways to order 4 people not A

*

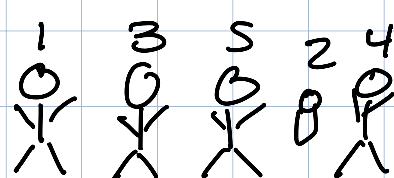
4 locations to put

Recognizing that each of you disjoint sets is the same

Example: How many ways to order portrait of 5 people if person 2 is a baby and must be on person 1's immediate right?



Good!



Bad?

treat 1/2 as same item

$n = 4$ $\quad$ $k = 4$

$P(4,4) = 4!$

Exercise:

1) How many passwords can be made of lowercase letters no longer than 5 characters? (partition)

length 1	26
2	26^2
3	26^3
4	26^4
5	26^5

$\boxed{26 + 26^2 + 26^3 + 26^4 + 26^5}$

order matters

reuse options	! reuse
n^k	$P(n,k)$

2) How many length 4 passwords (lowercase) start w/a or end w/b

(partition, but be careful!)

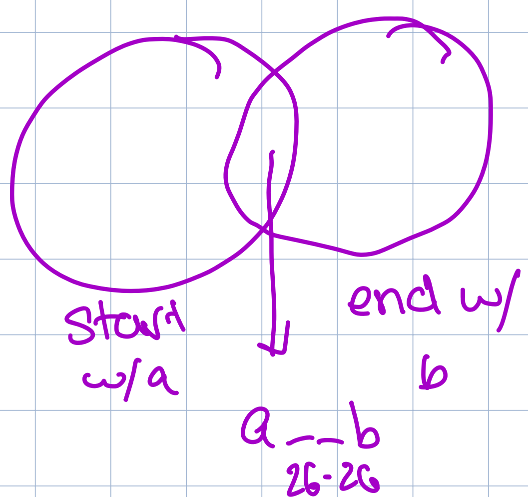
starts w/a

$\underline{a}\ \underline{26}\ \underline{26}\ \underline{26}$ $\quad 26^3$

ends w/ b

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{b} = 26^3$

$\boxed{(26^3 + 26^3) - 26^2}$



2)

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

$|U|$ = all passwords of length 10

26^{10}

$|I|$ = things that start w/ qwerty

q w e r t y _ _ _ _ 26^4

$|U| - |I| = \boxed{26^{10} - 26^4}$

3)

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
Assume:

- there are 10 Montague's (Romeo's family, excluding him) who could be in the photo
- there are 7 Capulets (Juliet's family, excluding her) who could be in the photo
- Romeo and Juliet are too busy dancing to be in any picture
- Montagues and Capulets won't get in the same photo (that whole Tybalt / Mercutio thing...)

(hint: partition, simplify a bit)

Photos of M | xor | Photo of C

$N = 10$ $K=3$ | $N=7$ $K=3$

$P(10,3) = \frac{10!}{7!}$ | $P(7,3) = \frac{7!}{4!}$

$$\boxed{\frac{10!}{7!} + \frac{7!}{4!}}$$

4) How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: ~~partition~~, ~~complement~~) complement → partition

3 1 2 5 6 — valid

2 1 4 7 3 — invalid

$|U| - |I|$

$|U| =$ $n=7$ $k=5$

$P(7,5)$

$$\left[\begin{array}{ccccc} 2 & 1 & \_ & \_ & \_ \\ \_ & 2 & 1 & \_ & \_ \\ \_ & \_ & 2 & 1 & \_ \\ \_ & \_ & \_ & 2 & 1 \end{array}\right] 4 \cdot P(5,3)$$

$$P(7,5) - 4 \cdot P(5,3)$$

Parent + Baby approach - tricky! Because we can exclude folks thus can't treat 2/1 as same person