CS1800 Discrete Structures	Prof Higger & Prof Hamlin
Fall 2024	Oct 2024

Exam1 (Practice)

Due: do-not-submit

This practice exam is longer than the real exam.

We hope you appreciate having extra practice problems, but know much of the material here is redundant with itself. (This is particularly true of the counting problems towards the end, which students tend to want a bit more practice with). The following instructions, counting formulas and logical identities are identical to your real exam. These rules aren't applicable for this practice exam here, but we include them in case you'd like to read ahead of time.

Exam Instructions:

- **Please write your name in the box on page 3 of the exam**
- Only the work on the front of each page containing a problem will be uploaded and graded.
- Please use the first two pages of the exam for all scratch work.
- This exam is a strictly individual effort, you may not view or share exam content (including discussing exam questions with other students) until grades have been released.
- Unless otherwise specified, be sure to show your detailed work to earn full credit
- You are welcome to use a calculator without any communication abilities (e.g. no cellphones, or wifi / bluetooth enabled calculators).
- To keep the exam fair to all students, no math-content related help will be offered to students. We are, however, happy to clarify any instructions but please don't expect any hints.
- Your work must be easiliy legible to be graded.
- When the final exam time has been called, please promptly turn in your work. Exams which are submitted late may be penalized 10 points per minute late (in practice we are not likely to apply this penalty so long as you stop working and get up to hand in your exam when time is called).

Reminders:

- Combinations: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$
- Permutations: $P(n,k) = \frac{n!}{(n-k)!}$
- Product Rule: n^k
- Ball-in-bins (or Stars & Bars): $\binom{n+k-1}{n-1}$

Logic and Set Identities:

The identities below are shown in the language of Boolean Algebra on the left (P, Q, R are Boolean variables) and Set Algebra on the right (A, B, C are subsets of some universal set U). This document uses the C superscript (A^C) instead of the bar notation ($\bar{A}$) to indicate the complement operation, they mean the same thing.

Associative Laws

(P ∨ Q) ∨ R = P ∨ (Q ∨ R)	$(A \cup B) \cup C = A \cup (B \cup C)$
(P ∧ Q) ∧ R = P ∧ (Q ∧ R)	$(A \cap B) \cap C = A \cap (B \cap C)$

Double Negation

¬ ¬ P = P	$(A^C)^C = A$

DeMorgan's Laws

¬(P ∨ Q) = ¬ P ∧ ¬ Q	$(A \cup B)^C = A^C \cap B^C$
¬(P ∧ Q) = ¬ P ∨ ¬ Q	$(A \cap B)^C = A^C \cup B^C$

Distributive Laws

P ∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R)	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R)	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Absorption Laws

P ∧ (P ∨ Q) = P	$A \cap (A \cup B) = A$
P ∨ (P ∧ Q) = P	$A \cup (A \cap B) = A$

Complement Laws

P ∨ ¬ P = T	$A \cup A^C = U$
P ∧ ¬ P = F	$A \cap A^C = \emptyset$

Idempotent Laws

P ∨ P = P	$A \cup A = A$
P ∧ P = P	$A \cap A = A$

Identity

False ∨ P = P	$\emptyset \cup A = A$
True ∧ P = P	$U \cap A = A$

Domination:

True ∨ P = True	$U \cup A = U$
False ∧ P = False	$\emptyset \cap A = \emptyset$

Clearly print below (to be read by a computer):

Name:	NUID:

Problem 1 Number Representation

Consider the addition of $(1111)_2$ and $(0101)_2$ in a 4-bit two's complement number system:

i Compute the addition in binary (your output must also be 4-bits).

ii Write the decimal equivalent of the operation above. For example, you might write $5 + 3 = 8$.

iii Is there an overflow in this operation? Explain why or why not in one sentence.

Problem 2 Base conversion

i Convert the number 753 from decimal to binary using Euclid's Division Algorithm (the fast method).

ii Convert the binary number $(10100101)_2$ to decimal. Show each of the powers of two to be added to reproduce the value.

iii Convert the decimal number 45263 to base 16 (hexadecimal) using Euclid's Division Method:

iv Solve for x in the equation below:
$(BFF)_{16} = (x)_{10}$

Problem 3 Logic: English to Logic

The predicates below are applicable to any candies x, y:
Using the predicates above, express each english statement with logic:

i Snickers bars have chocolate, but no nuts

`choc`(x)	candy x contains chocolate
`nut`(x)	candy x contains nuts
`pop`(x)	candy x is popular
`same_comp`(x, y)	candy x and y are made by the same company

ii all popular candies contain nuts

iii there is no candy which is both popular and doesn't contains nuts. (Hint: use $\neg\exists$ here)

iv By negating the existential quantifier and applying DeMorgan's law, write a logically equivilent form of the statement immediately above. Note that these manipulations may be applied to the english sentence or the logical expression, if you're stuck on one try the other.

v For every candy with nuts, there is another candy made by the same manufacturer which doesn't have nuts.

Problem 4 Logic: Truth Table

The headlights of a car turn on for either of the following two reasons:

- the manual switch is on
- the automatic switch is on and the car does not detect any light (i.e. it's dark outside)

1. Construct a truth table for H in terms of A, L, M.
   - A = 1 when automatic headlight switch is on
   - L = 1 when light detected by light sensor
   - M = 1 when manual headlight switch is on
   - H = 1 when headlights on

A	L	M	H

2. Write a logic expression[1] for H in terms of A, L, M.

[1] For example, a logical expression for X in terms of R and E is $X = R \wedge E$

Problem 5 Set Algebra

Simplify each of the following expressions by applying (and labelling) one law at time from logic_set_identities.pdf. Do not use the set difference operator in your simplifications. Note that the set U in the second item is the universal set, which includes all elements.

i $A \cap A$

ii $(A^C \cap B^C)^C \cap U$

iii $(A \cup A) \cap (B \cup A^C)$

Problem 6 Counting: Assorted

A course has 4 sections. Each contains 239, 243, 87 and 49 students respectively.

i If six students from the smallest section form a study group, how many different groups could there be?

ii How many different study groups can be formed from one student from each section? (Four total students in the group).

iii How many ways can a particular student, Sally, from the second section form a study group with two students from the first section (239 students)?

iv Sally is now willing to form a group with two students from either the first *or* third sections (potentially one from each), how many different groups can she form?

v Sally realizes that the two sections have different styles, so she now wishes to select the two students from either the first or third section, but not both, how many different groups can she form?

Problem 7 Principle Inclusion/Exclusion OS

There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

Problem 8 Generous Lee (Repeated from Rec5)

i Lee wants to hang 8 of his 20 photos in an ordered line on the wall above his desk. How many ways can he do this?

ii Lee wants to select 9 photos of his 20 to take on a trip. How many ways can he do this?

iii Lee has decided to give his collection of 20 unique photographs to his three children. In how many ways can he partition his photo collection among this three children where it may be that some children get no photographs?

iv Lee also has 500 dollars he wants to give his three children. In how many ways can he divide it among his three children? It may be the case that a child receives no money. Assume that Lee is distributing whole dollar bills to his children, each bill cannot be exchanged for coins.

v Lee's children will be upset if he doesn't give each of them at least 50 dollars. How many ways can Lee partition his money so his children won't be upset?

Problem 9 Deliveries (Repeated from Rec5)

A delivery driver has 10 **unique** packages to deliver.

i How many ways can the delivery driver select 6 of these packages to deliver?

ii How many ways can the delivery driver select 4 of these packages to discard while delivering the rest? Explain how this problem relates to the first subproblem.

iii How many different routes may the delivery truck drive to deliver 8 of the 10 packages? A route is an ordered sequence of destinations. You may assume each package may only be delivered to its unique destination.

iv Before leaving for the day, the delivery trucks are loaded with packages. How many ways can 120 **unique** packages be loaded into 10 delivery trucks where some trucks may have no packages?

v How many ways can 120 **identical** packages be loaded into 10 delivery trucks where some trucks may have no packages?

vi The delivery truck driver union is concerned that the workload is unequal. How many ways can 120 **identical** packages be loaded into 10 delivery trucks where each truck must have at least 5 packages?

vii How many ways can 120 **unique** packages be loaded into 10 delivery trucks where each truck must have the same number of packages?