Written Homework 09

Assigned: Thu 29 Mar 2018
Due: Wed 04 Apr 2018

Instructions:

• The assignment has to be uploaded to Blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• You may turn in work to Blackboard that is either handwritten and scanned, written in a word processor such as Word, or typeset in LaTeX. In the case of handwritten work, we may deduct points if the scan is upside down or the work is illegible.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

• We expect that you might study with friends and work out solutions to problems together, but you must write up your own solutions, in your own words.

• Some guidelines on collaboration: If you produce a solution together on a whiteboard, don’t simply copy it down afterwards. You must, on your own, write your own solution by hand. If someone explains an answer to you, do not write down their exact words; instead, on your own write up your solution afterwards. In short, your solution should be uniquely yours, and a product of your own understanding.

• If you collaborate with anyone, write their name on the first page of your assignment at the top.

Problem 1 [18 pts (3 points each)]: Six Sequences
For each of the sequences below, state whether it is arithmetic, quadratic, geometric, or neither. If the sequence is of one of the three types, then give a closed formula for its general term. Assume that indexing starts at 1. For example, the sequence

\[ a = (9, 13, 16, 20, 24, 28, \ldots) \]

is arithmetic with difference 4. Its general term is given by the formula \( a_n = 5 + 4n \) (for \( n \geq 1 \)).

i. \( 6, 18, 54, 162, 486, 1458, \ldots \)

ii. \( 20, 17, 14, 11, 8, 5, \ldots \)

iii. \( 12, -24, 48, -96, 192, -384, \ldots \)

iv. \( 5, -9, 13, -17, 21, -25, \ldots \)

v. \( 4, 12, 26, 46, 72, 104, \ldots \)

vi. \( 5^{20} \times 7^6, 5^{19} \times 7^7, 5^{18} \times 7^8, 5^{17} \times 7^9, 5^{16} \times 7^{10}, 5^{15} \times 7^{11}, \ldots \)
Problem 2 [20 pts (4 pts each)]: Five Sums
Use the formulas discussed in class to compute the following sums. Show your work.

i. \(301 + 302 + 303 + 304 + \ldots + 500\)

ii. \(143 + 146 + 149 + 152 + \ldots + 2471\)

iii. \(9 + 27 + 81 + 243 + \ldots + 59049\)

iv. \(4 - 12 + 36 - 108 + 324 - \ldots + 236,196\)

v. \(\sum_{k=201}^{400} (7k + 3)\)

Problem 3 [20 pts (4, 4, 12)]: Algorithms

i. How many comparisons does Insertion Sort do when sorting the array 4, 8, 6, 9, 2? Assume that when inserting a new element, we compare it from right to left with the previously inserted elements.

ii. How many comparisons does Merge do to merge arrays 1, 2, 8, 9, 10 and 3, 4, 5, 6, 7?

iii. Which is the largest value of \(n\) for which you can solve within one second a problem that requires \(T(n)\) operations, where each of these operations is carried out in \(10^{-9}\) seconds, with these functions \(T(n)\)?

1. \(\log_2(n)\)
2. \(n\)
3. \(n \log_2 n\)
4. \(n^2\)
5. \(2^n\)
6. \(n!\)