Written Homework 08

Assigned: Thu 22 March 2018
Due: Wed 28 March 2018

Instructions:

• The assignment has to be uploaded to Blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• You may turn in work to Blackboard that is either handwritten and scanned, written in a word processor such as Word, or typeset in LaTeX. In the case of handwritten work, we may deduct points if the scan is upside down or the work is illegible.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [25 pts (7,6,12)]: Bayes’ Rule

i. “Ten!” you hear called out from the apartment next door. You know they like to play *Settlers of Catan*, which uses two six-sided dice, and also *13th Age*, which uses one twenty-sided die. You think your neighbors play each game equally often, and that the roll must have been from one of these two games (and therefore must have been either two six-sided dice for *Settlers of Catan* or one twenty-sided die for *13th Age*). Use Bayes’ Rule to calculate the probability that they are playing *Settlers of Catan*. Give an exact probability (either a fraction or a decimal).

ii. But, come to think of it, your neighbors actually play *Settlers of Catan* three times as often as *13th Age*; that is, only one in four games are *13th Age*, while three in four are *Settlers of Catan*. Use these prior probabilities, instead of the equally probable priors you used in the preceding problem, to calculate the probability your neighbors are playing *Settlers of Catan*. Give an exact fraction or an answer to 2 decimal places.

iii. You hear two more rolls called out: “Twelve! Two!” Use everything you know, including all three rolls and the the prior probabilities of the two games mentioned in the previous step, to figure out the final probability that the neighbors are playing *Settlers of Catan*. Give your answer to two decimal places.
Problem 2 [25 pts (3, 5, 6, 6, 5)]: Entropy

An MPEG file encodes video partly by using the previous frame of video as a reference. Each “macroblock” of 16x16 pixels gets one “motion vector” associated with it that indicates where to look in the previous frame for a block of 16x16 pixels that could be used as a starting point for the new 16x16 block of pixels. Suppose for this problem that we aren’t interested in the magnitude of each motion vector - we want to encode, for each frame of video, just the rough direction of each motion vector. It’s either up, up-right, right, down-right, down, down-left, left, up-left, or no motion (9 possibilities).

i. If the “no motion” possibility occurs half the time, how many bits should we use to encode this possibility in an efficient encoding of these directions?

ii. Of the remaining possibilities, left movement occurs 1/8 the time, right movement occurs 1/8 the time, down and up each occur 1/16 of the time, and the diagonals each occur 1/32 of the time apiece. Calculate the minimum number of bits necessary to represent each direction, assuming we have one code per symbol.

iii. What is the entropy of this directional data, using the frequencies given in the preceding steps? (Up to 3 decimal places)

iv. Suppose instead of the frequencies just mentioned, we have more active video where all 9 symbols are equally likely. Calculate the entropy of the motion directions in this scenario. (Up to 2 decimal places)

v. Explain why it makes sense from a qualitative point of view that the number you obtained in the preceding step is larger or smaller than the entropy that you calculated before.

Problem 3 [10 pts]: Markov Chains

Suppose we model the stock market as a Markov chain where the major indices’ direction of motion on any given day – up, down, or stay roughly the same – depends on their direction of motion the previous day – up, down, or stay roughly the same. (From this point forward, we’ll just call the thing doing the movements “the market.”)

If it went up the previous day, it has a 0.6 chance of continuing to go up, a 0.3 chance of staying the same, and a 0.1 chance of going down.

If it stayed the same on the previous day, it has a 0.2 chance of going up, a 0.7 chance of staying the same, and a 0.1 chance of going down.

If it went down the previous day, it has a 0.4 chance of going up, a 0.4 chance of staying the same, and a 0.2 chance of continuing to go down.

Find the stationary distribution of this Markov chain, which represents the overall fraction of the time that the market goes up, stays the same, or goes down. You should find exact probabilities expressed as fractions.

(Note: It’s easy to make a mistake here, so when you find a solution, check it against your initial equations.)