Written Homework 07

Assigned: Thu 15 March 2018
Due: Wed 21 March 2018

Instructions:

• The assignment has to be uploaded to Blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• You may turn in work to Blackboard that is either handwritten and scanned, written in a word processor such as Word, or typeset in LaTeX. In the case of handwritten work, we may deduct points if the scan is upside down or the work is illegible.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [22 pts (9,9,2,2)]: Alice, Bob, and Chloe take a quiz

Alice, Bob, and Chloe are taking a multiple-choice quiz that consists of four questions, each with four answers to choose from. An answer grid thus has the form

<table>
<thead>
<tr>
<th></th>
<th>selection from four choices A,B,C,D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Each correct answer is scored 1 point, each incorrect answer scores −0.5 point and each unanswered question scores 0 point. The score on the quiz is the sum of the scores on the individual questions, except that scores below 0 are rounded up to 0. For example, answers evaluated as (correct, incorrect, incorrect, correct) would score 1 point, but answers evaluated as (correct, incorrect, incorrect, incorrect) would score (−0.5 which is converted to) 0.

Our three students know the answer to Question 1, as it was just reviewed in class; they will thus answer that question correctly. They, however, differ in strategies as to how questions 2, 3, 4 should be answered.

i. Alice decides to skip question 2 and answer questions 3 and 4 by randomly selecting answers.

1. What are Alice’s possible scores on the quiz?
2. How many different answer grids are possible for Alice?
3. What is the probability that Alice’s answer grid has no incorrect question?
4. What is the probability that Alice’s answer grid has exactly two incorrect questions?
5. Construct a table that assigns its probability to each possible quiz score.

<table>
<thead>
<tr>
<th>score</th>
<th>3</th>
<th>. . . .</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr[score]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. What is Alice’s expected score?

ii. Bob’s strategy is to answer each of questions 2 through 4 randomly.

1. What are Bob’s’s possible scores on the quiz?
2. How many different answer grids are possible for Bob?
3. What is the probability that Bob’s answer grid has no incorrect question?
4. What is the probability that Bob’s answer grid has exactly three incorrect questions?
5. Construct a table that assigns its probability to each possible quiz score.

<table>
<thead>
<tr>
<th>score</th>
<th>4</th>
<th>. . . .</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr[score]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. What is Bob’s expected score?

iii. Chloe’s strategy is to not answer questions 2 through 4 (while giving the answer she knows is correct for question 1). What is Chloe’s expected score?

iv. Which of the three strategies works best?

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**Problem 2** [22 pts (4,4,6,4,4)]: Colored Balls

An urn contains 15 balls: 10 white balls numbered 1 through 10 and 5 black balls numbered 1 through 5. We are simultaneously and randomly drawing 2 balls out of the 15.

i. Find the probability of event $A$: the two balls are white.

ii. Find the probability of event $B$: the two balls are odd.

iii. Are events $A$ and $B$ independent? Prove your answer.

iv. Let $X$ be the random variable whose value is the number of white balls in the drawing.

1. Write the probability distribution of $X$ in form of a table:

<table>
<thead>
<tr>
<th>Pr[$X=0$]</th>
<th>Pr[$X=1$]</th>
<th>Pr[$X=2$]</th>
</tr>
</thead>
</table>

2. Find the expected value $E[X]$.  

2
Problem 3 [16 pts (4 pts each)]: Distribution of balls in urns

In this problem, we have four distinguishable urns named A, B, C, D and four identical balls to be placed in these urns. We first calculate the number of different placements that are possible and then discuss the probabilities of these placements.

i. In how many ways can one distribute the four identical balls among urns A, B, C, and D?

ii. Suppose that the four balls are randomly thrown at the urns and that each ball has an equal chance to land in any of the urns. What is the probability that each urn receives a ball? Hint: paint the balls with numbers 1, 2, 3, 4 to make them distinguishable. Then each throw of the balls can be represented by a string of four letters. For example, string B A D D indicates that ball 1 landed in urn B, ball 2 in urn A, and balls 3 and 4 in urn D.

iii. Is the probability of landing one ball in each urn equal to the ratio of the number of placements with one ball per urn (this is 1) over the number you calculated in part (i)? What can you infer from this: are the placements equally likely?

iv. You receive $1 for each ball landing in urn A, $2 for each ball in urn B, $3 for each ball in urn 3, and $4 for each ball in urn D. Use linearity of expectation to calculate your expected gain. (Explain how and where linearity of expectation was used in your calculation.)