Written Homework 01

Assigned: Wed 17 Jan 2018
Due: Wed 24 Jan 2018

Instructions:

- The assignment has to be uploaded to Blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

- We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

- You may turn in work to Blackboard that is either handwritten and scanned, written in a word processor such as Word, or typeset in LaTeX. In the case of handwritten work, we may deduct points if the scan is upside down or the work is illegible.

- To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [25 pts (3,3,2,3,5,4,5)]: Base conversions

i. By hand, convert the binary number 01001011 from binary to decimal, showing the powers of two that must be added to produce the correct solution.

Solution: \(64+8+2+1 = 75\)

ii. Convert the number from the previous part to hexadecimal, and show mathematically how each digit multiplied by a power of 16 produces the same decimal number that you found in part (i).

Solution: \(4B; 4*16 + 11*1 = 64+11 = 75\)

iii. Consider the binary numbers obtained by separating the 8-bit binary number from part (i) into the two 4-bit binary numbers 0100 and 1011. What is the relationship between these numbers and the hexadecimal digits?

Solution: These are the decimal numbers 4 and 11 which can be written as 4 and B in hex. We thus know that 0100 1011 is 4B in hex.

iv. Convert (by hand) the hexadecimal number \(BEAD_{16}\) to sixteen digits of binary, using whatever method you like. Show and explain your work.

Solution: Converting each hex digit to binary individually is the easiest way to do this. B = 1011, E = 1110, A = 1010, D = 1101, so the complete binary is 1011 1110 1010 1101.
v. Use a method similar to the shortcut for converting binary to hexadecimal to convert the base 3 number \((212201211100002)_3\) to base 9.

\textit{Solution:} Separating the number into groups of two digits at a time gives \(21\ 22\ 01\ 20\ 12\ 11\ 10\ 00\ 02\) which is \((781654302)_9\).

vi. Convert \((571846)_9\) to base 3 using a similar trick.

\textit{Solution:} Applying the same logic going the other direction gives \(12\ 21\ 01\ 22\ 11\ 20\) as the two digit sequences, or \((122101221120)_3\).

vii. Write the binary number \(11011011\) in base 32. Assume digits beyond 15 continue to use the normal alphabet (\(g=16,\ h=17,\) etc.).

\textit{Solution:} 6R. You can actually use the same trick as hex here, turning every 5 digits into a base 32 digit.

Problem 2 [20 pts (4 pts each)]: Two’s Complement

i. Convert the decimal numbers -104 and -18 to two’s complement, assuming an eight bit two’s complement representation. Show your work, including how you get from the binary for positive integers 104 and 18 to their negative counterparts.

\textit{Solution:} 104 = 01101000, so -104 = 10010111+1 = 10011000.

18 = 00010010, so -18 = 11101101 + 1 = 11101110.

ii. Sum the numbers you produced in the preceding question using binary addition. (You must show where you carry for full credit.) Verify that the result is equal to the binary for -122 by converting to a positive integer.

\textit{Solution:}

\[
\begin{array}{c}
1 \\
1 \\
1 \\
+ \\
1 \\
\hline
1
\end{array}
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}
\]

10000110 flipped is 01111001; add 1 to get 01111010 = 64+32+16+8+2 = 122. So this is the binary for -122, as it should be.

iii. If you are using 64-bit signed numbers (two’s complement), what is the sum of the number represented by 64 1 bits (1111...1) and the number represented by 56 0 bits followed by 8 1 bits (000...011111111)? (Give the answer in decimal.)

\textit{Solution:} The all one’s number must be equal to -1, since adding one to it would produce 0. The other number is 255, one less than \(2^8\). So their sum must be 254.
iv. Suppose I am using an ancient computer architecture that uses 10-bit numbers with a two’s complement representation of negative numbers. What is the smallest positive number that, when added to itself, produces overflow (a result with a negative interpretation)? Give the answer in both binary and decimal.

**Solution:** With 10 bits we have \(2^{10} = 1024\) strings to represent 1024 numbers. One half is used to represent numbers in range 0..511, the other half is for numbers in range -1..-512. Overflow when computing \(2x\) for \(0 \leq x \leq 511\) occurs when \(2x \geq 512\), that is, when \(x \geq 256\). The smallest such number is \(x = 256\).

Another view is that multiplying by 2 is just adding a zero at the right end. The first negative number is 10 0000 0000 which is the double of 01 0000 0000. This number is just \(2^8 = 256\) as obtained above.

v. Generalize your answer to the preceding question to the \(n\)-bit case: with \(n\) bits, what number is the smallest possible positive number that, when added to itself, overflows (produces a number with a negative interpretation)? Give your answer in decimal, in terms of \(n\).

**Solution:** The smallest possible number that overflows when doubled is \(2^{n-2}\).

Problem 3 [15 pts (8, 7)]: Locating defective parts.

i. Your local Be-Computer store has received 10 containers with 100 bePads in each container. Each bePad is marked to identify the container it is from. Other than that, all bePads are identical and weigh exactly one pound, except for those in one container that have a manufacturing defect. Those defective bePads weigh exactly 17 ounces each. You are asked to identify the bad container. Of course, you could weigh one bePad from each container until your scale measures 17 ounces. But this process may take 10 weighings and you are asked to instead use only one single weighing. The good news is that your scale is an industrial grade (single platform) scale and you can place any number of bePads on its platform. How then do you identify the problematic container by finding the weight of just one collection of bePads selected from various containers?

**Solution:** Select one bePad from container 1, two bePads from container 2, \ldots, ten bePads from container 10 and weigh this collection. If no bePads been defective, the total weight would have been \(1 + 2 + 3 + \ldots + 10 = 55\) pounds. But some container, say container \(n\), has defective bePads. This container contributes \(n\) pounds + \(n\) ounces, instead of just \(n\) pounds. That is, the container number \(n\) is given by

\[
 n = \text{measured weight} - 55
\]

expressed in ounces.

ii. Having heard of your success with single weighings, the next town’s Be-Computer store calls you for help. They have received 7 containers (with 100 bePads in each) and know that some of these containers may be coming from the lot of defective 17-ounce bePads. However, they don’t know how many containers are bad. It could be any number, including all or none. How can you in a single weighing identify all the bad containers?

**Solution:** Select powers of two, that is, from container 1: 1 bePad, from container 2: 2 bePads, from container 3: 4 bePads, \ldots, from container 7: 64 bePads. If no container is bad,
the weight of this collection is $1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$ pounds. If container $n$ is bad, it contributes $1 \times 2^{n-1}$ additional ounces. That is, to identify the bad containers, you find the binary representation of the number

$$\text{measured weight} - 127$$

expressed in ounces. The 1’s in this representation indicate the bad containers. For example, a binary 0010101 would indicate that containers 1, 3, and 5 are bad.