Written Homework 03

Assigned: Fri 06 Oct 2017
Due: Fri 20 Oct 2017

Instructions:

• The assignment has to be uploaded to Blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• You may turn in work to Blackboard that is either handwritten and scanned, written in a word processor such as Word, or typeset in LaTeX. In the case of handwritten work, we may deduct points if the scan is upside down or the work is illegible.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [20 pts (4,12,4)]: Set Union and Intersection

Three sets are given:

A = \{natural number multiples of 2, no more than 300\}
B = \{natural number multiples of 3, no more than 300\}
C = \{natural number multiples of 5, no more than 300\}

A. Define these sets formally with set-builder notation.

B. Compute the size of the union of these three sets.

C. Compute the size of the intersection of these three sets.

Problem 2 [20 pts (5,5,10)]: Count Possibilities with Bits

A. How many natural numbers smaller than 1024 are the sum of four different powers of 2?

hint. Think of this problem in binary (bits). What does it mean for a number to be the sum of four different powers of 2?

B. How many natural numbers between 512 and 1024 are the sum of four different powers of 2?
C. How many natural numbers smaller than 1000 are the sum of four different powers of 2?

Problem 3 [20 pts (7,13)]: Interleave Preserving Relative Order

A. Given the sequence (a,b,c,d,e,f,g), in how many ways can be organized into 5 bins on line such that the literals remain in the same order?
Example valid arrangements are
   (ab) () (cde) (f) (g)
   () (a) (bc) (d) (efg)
   (a) (bc) (defg) () ()

B. Two sorted sequences lengths 9 and 7 are given: (1,2,3,...9) and (a,b,c,d,e,f,g). We want to interleave them into a sequence of length 16 such that numbers 1-9 remain in relative order, and also literals a-g remain in relative order. How many ways are there to do this?
Example valid sequences are
   1a2bc34d56efg789
   12345abc678de9fg
   a1bcdef23456789g

Problem 4 [20 pts (7,13)]: Counting Cumulative Hits

Set M = \{1,2,3,4,5,6,7\}.

A. If we allow each student to pick a number from M, and every number can be picked at most twice, what is the maximum number of students that can pick numbers?

B. What is the max number of 4-element-subsets of M we can select, such that intersection of any 3 of them is empty?

Problem 5 [20 pts (7,13)]: Passwords with non-Neighboring Digits

A. List all binary 6-bit sequences that do not have two “1” bits next to each other. Organize them into groups by the total number of “1” bits.

B. Valid passwords of length 6 can use the 10 digits and the 26 capital letters in any order, with the condition that two digits cannot be next to each other. How many such passwords are there?