Last time
- Finish Expectation
  - Variance / standard deviation
- Start Entropy

Today
- Finish Entropy
- Conditional Prob. & Bayes Law

Next time
- Markov Chains & Page Rank

Announcements
Regrades → Kevin Gold
\[ p_2 = \left( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64} \right) \]

\[ \begin{array}{cccccccc}
0 & 10 & 110 & 1110 & 11110 & 111110 & 1111110 & 1111111 \\
\end{array} \]

\[ \text{BPC} = \sum_{i} l_i \cdot p_i \]

\[ = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{16} + 6 \cdot \frac{1}{64} \]

\[ = 2 \]

\[ \begin{array}{cc}
\frac{p_i}{l_i} & l_i \\
\frac{1}{2} & 1 \\
\frac{1}{4} & 2 \\
\frac{1}{8} & 3 \\
\frac{1}{16} & 4 \\
\frac{1}{64} & 6 \\
\end{array} \]

\[ p_i = \frac{1}{2^k} \]

\[ l_i = k \Rightarrow p_i = \frac{1}{2^l} \]

\[ 2^{l_i} = \frac{1}{p_i} \]

\[ l_i = \log_2 \frac{1}{p_i} \]
Entropy

Q: \( P = (p_1, p_2, p_3, \ldots) \)

\( l_i = \log_2 \frac{1}{p_i} \)

What is expected code length?

\( BPC = \sum_i p_i \cdot l_i \)

\( = \sum_i p_i \cdot \log_2 \frac{1}{p_i} \)

\( H(P) = \sum_i p_i \cdot \log_2 \frac{1}{p_i} \)

Facts

1. Entropy is the compression limit.
2. Entropy is essential to bound an ability to communicate in presence of noise.
3. Fundamental measure of randomness.
4. Cryptography
Conditional Probability

\[ \Pr (H \cap E) = \Pr (H) \cdot \Pr (E | H) \]

\[ \Rightarrow \Pr (E | H) = \frac{\Pr (H \cap E)}{\Pr (H)} \]

\[ = \Pr (E) \cdot \Pr (H | E) \]

\[ \Rightarrow \Pr (H | E) = \frac{\Pr (H \cap E)}{\Pr (E)} \]

Bayes Law:

\[ \Pr (E) \cdot \Pr (H | E) = \Pr (H \cap E) = \Pr (H) \cdot \Pr (E | H) \]

\[ \Pr (H | E) = \frac{\Pr (E | H) \cdot \Pr (H)}{\Pr (E)} \]

\( H = \text{hypothesis: do have Zika?} \)

\( E = \text{evidence: did blood test come back pos?} \)