Last time
- Finished Probability Examples
- Started Expectation

Today
- Expectation & Variance

Next time
- Entropy
Underlying probability measure induces distribution over range of random variables.

\[ \mathbb{P}(A) = \mathbb{P}(B) = \mathbb{P}(C) = \frac{1}{3} \]

\[ \mathbb{E}[X_1] = \sum_x x \cdot \mathbb{P} \{ X_1 = x \} = 25000 \cdot \frac{1}{3} + 35000 \cdot \frac{2}{3} = 31666.67 \]

\[ \mathbb{E}[X_2] = \sum_x x \cdot \mathbb{P} \{ X_2 = x \} = 2014 \cdot \frac{2}{3} + 2017 \cdot \frac{1}{3} = 2015 \]

\[ X_1 = \text{purchase price} \]
range \((x_1) = \{25000, 35000\} \]
ind. dist. \[ D(25000) = \mathbb{P} \{ X_1 = 25000 \} = \frac{1}{3} \]
\[ D(35000) = \mathbb{P} \{ X_1 = 35000 \} = \frac{2}{3} \]

\[ \mathbb{E}[X_1] = \sum_x x \cdot \mathbb{P} \{ X_1 = x \} = 25000 \cdot \frac{1}{3} + 35000 \cdot \frac{2}{3} = 31666.67 \]

\[ X_2 = \text{model year} \]
range \((x_2) = \{2014, 2017\} \]
ind. dist. \[ D(2014) = \mathbb{P} \{ X_2 = 2014 \} = \frac{2}{3} \]
\[ D(2017) = \mathbb{P} \{ X_2 = 2017 \} = \frac{1}{3} \]

\[ \mathbb{E}[X_2] = \sum_x x \cdot \mathbb{P} \{ X_2 = x \} = 2014 \cdot \frac{2}{3} + 2017 \cdot \frac{1}{3} = 2015 \]
1. Roll one fair six-sided die \( \{1, 2, 3, 4, 5, 6\} \)

- \( X = \) value of die face
- \( X(1) = 1 \)
- \( X(2) = 2 \)
- \( X(6) = 6 \)

What is \( E[X] \)?

Prob. meas.: \( p(\cdot) = p(\cdot) = p(\cdot) = \ldots = p(\cdot) = \frac{1}{6} \)

\[ \Pr\{X = 1\} = \Pr\{X = 2\} = \ldots = \Pr\{X = 6\} = \frac{1}{6} \]

\( \Pr\{X = 1\} = \Pr\{X = 2\} + \ldots + \Pr\{X = 6\} = \frac{1}{6} \)

\[ E[X] = \sum_{x} x \cdot \Pr\{X = x\} = \sum_{i=1}^{6} i \cdot \Pr\{X = i\} = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \ldots + 6 \cdot \frac{1}{6} \]

\[ = \frac{1}{6} \cdot (1 + 2 + 3 + 4 + 5 + 6) = 3.5 \]

\[ E[X] = \sum_{\omega \in \Omega} x(\omega) \cdot p(\omega) = \sum_{\omega \in \Omega} x(\cdot) \cdot \frac{1}{6} + x(\cdot) \cdot \frac{1}{6} + \ldots + x(\cdot) \cdot \frac{1}{6} \]

\[ = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \ldots + 6 \cdot \frac{1}{6} = 3.5 \]
2) Roll two fair six-sided die
   \( X = \) sum of values of die faces
   What is \( E\{X\} \)

\[
E\{X\} = \sum_{x} x \cdot P_r\{X = x\}
\]

\[
\begin{align*}
X = 2 & \quad P_r\{X = 2\} = \frac{1}{36} \\
X = 3 & \quad P_r\{X = 3\} = \frac{2}{36} \\
X = 4 & \quad P_r\{X = 4\} = \frac{3}{36} \\
\vdots & \\
X = 12 & \quad P_r\{X = 12\} = \frac{1}{36}
\end{align*}
\]

\[
E\{X\} = \sum_{x} x \cdot P_r\{X = x\}
\]

\[
= 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \cdots + 12 \cdot \frac{1}{36}
\]

\[
= \frac{1}{36} \left( 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 \cdots + 12 \cdot 1 \right)
\]

**Linearity of Expectation**

\( X_1 = \) first die value
\( X_2 = \) second die value

\[
E\{X_1 + X_2\} = E\{X_1\} + E\{X_2\}
\]

\[
= 3.5 + 3.5 = 7
\]
Pay $6 to play
Roll a 6-sided die
Payout is sum of die faces except if doubles, then 0.

X = winnings (profit)

\[ E(X) = \sum_{x} x \cdot \text{Pr}(X = x) \]

\[ x = -6 \quad \text{Pr}(X = -6) = \frac{6}{36} = \frac{1}{6} \]
\[ x = -3 \quad \text{Pr}(X = -3) = \frac{3}{36} = \frac{1}{18} \]
\[ x = +5 \quad \text{Pr}(X = 5) = \frac{3}{36} = \frac{1}{18} \]

\[ E(X) = \sum_{x} x \cdot \text{Pr}(X = x) \]
\[ = (-6) \cdot \frac{6}{36} + (-3) \cdot \frac{3}{36} + 5 \cdot \frac{3}{36} \]
\[ = -0.166 \]

lose 16.6¢ on play or about 2.8%