Last time
- Finish counting
  - examples

Today
- Start Probability

Next time
- Continue probability
  - examples
Probability

- Random experiment
- generates outcomes: \( W \in \Omega \)
- Sample space: set of all possible outcomes
- Event: subset of sample space

- \( p: \Omega \to \mathbb{R} \) probability measure
  - \( 0 \leq p(w) \leq 1 \quad \forall w \in \Omega \)
  - \( \sum_{w \in \Omega} p(w) = 1 \)

Example

- Roll a fair six-sided die rolled a 5
  - \( \xi \{1, 2, 3, 4, 5, 6\} = \Omega \)
  - \( E_1 = "even" = \{2, 4, 6\} \)
  - \( E_2 = "2\ 3" = \{3, 4, 5, 6\} \)
  - \( p(1) = p(2) = \ldots = p(6) = \frac{1}{6} \)

\[
p(E) = \sum_{w \in E} p(w)
\]

\( E_1 \):

\[
p(E_1) = p(2) + p(4) + p(6)
\]

\( E_2 \):

\[
p(E_2) = \frac{1}{6} \quad \forall w \in \Omega
\]

\[
p(E) = \frac{|E_1|}{|\Omega|} \quad \text{e.g., } p(E_1) = \frac{3}{6}
\]
Examines

1. Roll one fair die
   \[ \mathcal{E} = \{1, 2, 3, 4, 5, 6\} \]

2. Roll two fair dice
   \[ \mathcal{E} = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (6,1), (6,2), (6,3), (6,4) \} = \mathcal{E}_1 \times \mathcal{E}_2 \]
   \[ |\mathcal{E}| = 36 \]

   \[ E_1 = \text{total } \geq 7 \]
   \[ p(E_1) = \frac{|E_1|}{|\mathcal{E}|} = \frac{6}{36} = \frac{1}{6} \]

   \[ E_2 = \text{total } \geq 8 \]
   \[ |E_2| = \frac{1+2+3+4+5+6+7+8+9}{12} = 10 \]
   \[ p(E_2) = \frac{|E_2|}{|\mathcal{E}|} = \frac{10}{36} = \frac{5}{18} \]
Candy:

- Standard deck of cards: 4 suits (Hearts, Diamonds, Clubs, Spades) within each suit 2, 3, 4, ..., 10, J, Q, K, A
- 13 total per suit

Rand. Exp.: Draw one card from deck

- Sample space: \( S = \{2H, 3H, ..., AH, 2D, 3D, ..., AS\} \) face cards per suit
  \( |S| = 52 \)
  \( 3 \times 4 \text{ suits} \)

- \( E_1 = \) face card (Jacks, Queens, Kings) \( |E_1| = 3 \times 4 = 12 \)
  \[
p(E_1) = \frac{|E_1|}{|S|} = \frac{12}{52} = \frac{3}{13}
  \]

- \( E_2 = \) card is between 2 & 10 (a number) \( |E_2| = 9 \times 4 = 36 \)
  \[
p(E_2) = \frac{|E_2|}{|S|} = \frac{36}{52} = \frac{9}{13}
  \]
**Urn Problems**: 15 red balls, 10 blue balls

- Rand. Exp.: draw one ball from urn

\[ \mathcal{D} = \{ R_1, R_2, \ldots, R_{15}, B_1, B_2, \ldots, B_{10} \} \quad |\mathcal{D}| = 25 \]

- \( E_1 = \text{red} \) \( |E_1| = 15 \)

\[ \Pr(E_1) = \frac{|E_1|}{|\mathcal{D}|} = \frac{15}{25} = \frac{3}{5} \]

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- Rand. exp.: draw 3 balls at once (sampling w/o replacement)

\[ \mathcal{S} = \{ \{ R_1, R_2, R_3 \}, \{ R_1, R_2, R_4 \}, \ldots, \{ R_1, R_9, B \}, \{ B_8, B_9, B_{10} \} \} \]

\[ |\mathcal{S}| = \binom{25}{3} = 2300 \]

- \( E_1 = \text{all reds} \) \( |E_1| = \binom{15}{3} \)

\[ \Pr(E_1) = \frac{|E_1|}{|\mathcal{S}|} = \frac{\binom{15}{3}}{2300} = \frac{455}{2300} = \frac{91}{460} \approx 19.8\% \]
$E_2 = \text{2 red, 1 blue}$

$|E_2| = \binom{15}{2} \cdot \binom{10}{1} = 1050$

\[ P(E_2) = \frac{\binom{15}{2} \cdot \binom{10}{1}}{\binom{25}{3}} = \frac{1050}{2300} \approx 45.7\% \]