Written Homework 06

Assigned: Wed 30 March 2016
Due: Wed 13 April 2016

Instructions:

• The assignment has to be uploaded to blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them by hand.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [52 pts, 13 pts each]: Induction

In each of the following problems, prove by induction. You must show all the steps.

i. Prove that $2^0 + 2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 1$ for all nonnegative integers $n$.

ii. Prove that 6 divides $n^3 - n$, for all $n \in \mathbb{N}$.

iii. Use induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for all nonnegative integers $n$.

iv. Water balloon fight. An odd number of people stand in a yard at mutually distinct distances. At the same time, each person throws a water balloon at their nearest neighbor, hitting this person. Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a water balloon.

Problem 2 [48 pts, (16, 16, 16)]: Recurrences

In each of the following problems, solve the recurrence using the method described in class and in the text. You must show your work.

Assume a base case of $T(1) = 1$. As part of your solution, you will need to establish a pattern for what the recurrence looks like after the $k$-th iteration. Your solutions may involve $n$ raised to a power and/or logarithms of $n$. For example, a solution of the form $8^{\log_2 n}$ is unacceptable; this should be simplified as $n^{\log_2 8} = n^3$. Express your final answer in terms of the average case performance, eg., $\Theta(n)$.
i. \( T(n) = 2T(n/3) + n. \)

ii. \( T(n) = 3T(n/3) + n. \)

iii. \( T(n) = 4T(n/3) + n. \)