Written Homework 4

Assigned: Wed 24 Feb 2016
Due: Wed 16 Mar 2016

Instructions:

• The assignment has to be uploaded to blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

• We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

• We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them by hand.

• To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [16 pts, (8 pts each)]: Choosing Students
A student is chosen at random from a class of 80 students that has 20 honor students, 30 athletes and 40 that are neither honor students nor athletes.

i. What is the probability that the student selected is an athlete given that they are an honor student?

ii. What is the probability that the student selected is an honors student given that they are an athlete?

Problem 2 [16 pts (5, 5, 6)]: Standing in Line
Consider the problem of a group of antagonistic people posing together for a picture designed to show their collaborative spirit.

i. Given a group of eight Democrats and five Republican politicians, how many ways are there to choose two Democrats and two Republicans for a photo op demonstrating bipartisanship? Give your answer as a number.

ii. Since the first picture, one Independent has been added to the group. Now, they wish to take a picture of the whole group together. How many ways are there for eight Democrats, five Republicans, and one Independent to stand in a line so that no Democrat and Republican need to stand next to each other? Give your answer as a number.
iii. How many ways are there for eight Democrats and five Republicans to stand in a line so that
no two Republicans stand next to each other? Give your answer as a number.

Problem 3 [24 pts (6,6,12)]: Solving an Art Heist

Five valuable paintings have been stolen by a lone thief from a distinguished museum in Boston.
You are the chief investigator assigned to resolve this crime.

Show all your work for each of the following parts. For parts (i) and (ii), you need not complete
all the calculations to yield a single number, but simplify each answer to the extent possible; for
instance, you may leave your answers in terms of expressions involving factorials. For part (iii),
complete the calculation, using a software calculator, Excel, or other means.

i. You have learned from reliable sources that each of the five paintings has been sold to a
different private collector in the country. You have narrowed down the list of possible collectors
who could have purchased any of these paintings to 100. Unfortunately, you have the resources
to conduct searches of only 30 of these collectors. If you select the 30 collectors uniformly
at random from the list of 100 and search their complete collections, what is the probability
that you will find at least one of the stolen paintings?

ii. Continuing from the above question, what is the probability that you will find at least four
of the five paintings?

iii. You have a database of 25,000 potential criminals. Based on your past analysis you estimate
that the probability that this database includes the art thief is 0.1. In a stroke of luck, a
thorough search of the crime scene has given you what you had been yearning for – a DNA
sample of the thief. You compare the sample with a database that you have of 25,000 suspects.
And lo and behold, a match is found! You are well aware that DNA matches are not always
perfect: if you pick two different persons at random, the chance that their DNA samples
would match using the current testing techniques is 1 in 10,000. What is the probability that
the database includes the art thief, given that a DNA match has been found?

Problem 4 [24 pts (4,6,6,8)]: Air Traffic Control

You are an Air Traffic Controller at a busy airport, trying to schedule a number of flights for
takeoff. At this moment, there are five American Airlines flights, two Delta Airlines flights, and
three United Airlines flights, all waiting to leave their respective gates and take off from Runway
1. Only one flight can take off from the runway at any instant. The following table lists all the
waiting flights with their destinations.

<table>
<thead>
<tr>
<th>Airline</th>
<th>Flight Number</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>AA025</td>
<td>Los Angeles</td>
</tr>
<tr>
<td>American</td>
<td>AA145</td>
<td>Los Angeles</td>
</tr>
<tr>
<td>American</td>
<td>AA425</td>
<td>Chicago</td>
</tr>
<tr>
<td>American</td>
<td>AA6175</td>
<td>London</td>
</tr>
<tr>
<td>American</td>
<td>AA4377</td>
<td>New York</td>
</tr>
<tr>
<td>Delta</td>
<td>DL365</td>
<td>Atlanta</td>
</tr>
<tr>
<td>Delta</td>
<td>DL557</td>
<td>Detroit</td>
</tr>
<tr>
<td>United</td>
<td>UA201</td>
<td>Washington DC</td>
</tr>
<tr>
<td>United</td>
<td>UA3263</td>
<td>Newark</td>
</tr>
<tr>
<td>United</td>
<td>UA163</td>
<td>Los Angeles</td>
</tr>
</tbody>
</table>
Show all your work for each of the following parts. For each part, you need not complete all the calculations to yield a single number, but simplify each answer to the extent possible; for instance, you may leave your answers in terms of expressions involving factorials.

i. In how many ways can you schedule these flight takeoffs?

ii. Suppose that United flights have the highest priority, and hence should be scheduled first. In how many ways can you schedule these flight takeoffs so that United flights are all scheduled before others?

iii. The list of flights includes three flights to Los Angeles – AA025, AA145, and UA163. You need to ensure that the takeoffs of the three Los Angeles flights will be separated by at least one other flight between them (to ensure that they are not too close to each other in their takeoff flight paths). How many ways can you schedule the flight takeoffs to guarantee this condition?

iv. A schedule is said to be fair if at any time, the following condition holds for every airline: either all the flights of the airline have taken off, or the number of takeoffs for any other airline is at most one more than the number of takeoffs for this airline. For example, the following two schedules are fair

```
AA025  DL365  UA3263  UA163  AA425  DL557  AA6175  UA201  AA4377  AA145
DL365  UA163  AA025  UA3263  AA425  DL557  UA201  AA4377  AA145  AA6175
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One can check that at each point in the above schedules, the desired condition holds for every airline.

The following schedule, however, is not fair.

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AA025  UA3263  DL365  DL557  UA163  AA425  AA6175  AA4377  UA201  AA145
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In the schedule above, consider the instant right after eight flights have taken off. At this point, American has had four takeoffs while United has had only two even though there is one more United flight waiting to take off. The following schedule is also not fair.

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DL365  UA163  AA4377  UA3263  AA425  UA201  DL557  AA025  AA145  AA6175
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In the above schedule, consider the instant right after six flights have taken off. At this instant, United has had three takeoffs while Delta has had only one even though there is one more Delta flight waiting to take off.

How many fair schedules are there?

**Problem 5 [20 pts; (2,5,5,4,4))]: The Fencing Tournament**

Your little brother has been learning to fence and will compete in his first tournament next week. In order to be a properly supportive big sibling, you’re learning a bit about fencing tournaments and applying your knowledge of counting and probability.
A fencing tournament begins with "pools" where fencers are divided into groups of seven (a pool) and fence all the members of their pool in round-robin fashion (six bouts for each fencer). During a pool bout, the first fencer to five points wins, with each fencer finishing with between 0 and 5 points. Each of those pool bouts could end in a W or a L for your brother (there are no ties).

For each of these parts, show all your work.

i. Just counting wins and losses, how many different pool outcomes are there for your brother if the order of wins and losses matters?

ii. How many different ways are there for him to win at least three bouts? Assuming all outcomes are equally likely, what is the probability that he will win at least three bouts?

iii. If he starts out a bit slow and loses his first bout, how many different ways are there for him to win at least three bouts? Again assuming all outcomes are equally likely, what is the probability that he will win at least three bouts after losing his first?

iv. Your brother’s ranking after pools will also be determined by how many more touches he scored than were scored against him (his "indicator"). If he wins a bout, he could have scored between 1 and 5 more touches than his opponent; if he loses, he could have between 1 and 5 more touches scored against him. A fencer’s indicator can range between 30 (if he wins all bouts and his opponents never score a point) and -30 (if he loses all bouts and he never scores a point). What is the distribution of possible indicators and how many different outcomes (counting all combinations) is there for someone who wins all of their six bouts?

v. Assuming all outcomes are equally likely, what is the probability that someone with all wins will have a perfect indicator of 30?