Written Homework 05

Assigned: Nov 9 2016
Due: Nov 21 2016

Instructions:

- The assignment has to be uploaded to Blackboard by the due date. No assignment will be accepted after 11:59pm on that day.

- We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

- We require that all homework submissions be neat, organized, and legible. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions, or scan handwritten work if it is legible. All work must be submitted in PDF format. Points may be deducted if your work is in a different format or unreadable.

- To get full credit, show intermediate steps leading to your answers, throughout.

Problem 1 [30 pts (6 each)]: Sequences
For each of the following lists of integers, give a simple formula that generates the terms of an integer sequence that begins with the given list. Start the number of your list elements at 1 (i.e., start with n=1). Also indicate whether the sequence is “arithmetic,” “geometric,” “quadratic,” or none of these.

i. 7, 10, 13, 16, 19, 22, . . .

ii. 2, 10, 50, 250, 1250, . . .

iii. 3, -6, 12, -24, 48, -96, . . .

iv. 3, 6, 13, 24, 39 . . .

v. 2, 4, 12, 48, 240, 1440 . . .

Problem 2 [32 pts (8 each)]: Sums
i. Evaluate the following sum. You must show your work, and your final answer should be a single integer.
\[ \sum_{k=5}^{100} 4k = \]
ii. Derive a formula in terms of $n$ for the following sum. You must show your work, and your final formula should only contain $n$ and integers (but not $k$).

$$\sum_{k=5}^{n} 4k =$$

iii. Derive a formula in terms of $a$ and $b$ for the following sum. You must show your work, and your final formula should only contain $a$, $b$, and integers (but not $k$).

$$\sum_{k=5}^{100} a \cdot b^k =$$

iv. Some games have players roll two differently colored ten-sided dice, numbered 0 through 9, to randomly generate the two digits of a number evenly distributed between 0 (00) and 99. Use Gauss’s trick to calculate the expected value of such a roll.

Problem 3 [8 pts, 4 each]: Growth of Sequences

Assume that the population of the United States on January 1, 2016 was 322,761,807 and it is growing at a rate of 0.7% per year.

i. Give a formula for the population of the United States on January 1 $n$ years from now.

ii. What does this formula predict the population of the United States will be in 2086?

Problem 4 [30 pts, 6 pts each]: Comparisons of Functions

Your software company is considering using various algorithms that have different running times for rendering to the screen. The “target platform” you expect your user to have is a laptop with a 1.5 GHz processor, meaning that the processor can handle 1,500,000,000 instructions per second. The algorithms must do some work for every pixel on the screen, and your target device has 1440 × 900 resolution, meaning there are $1440 \times 900 = 1,296,000$ pixels in all to handle. The algorithm must handle all the pixels at 30 frames a second, meaning the algorithm has to finish processing all the pixels using no more than $1,500,000,000/30 = 50,000,000$ instructions.

Below are the functions indicating each algorithm’s number of instructions executed as a function of the input size (here, number of pixels). For each: (a) identify whether it is logarithmic, linear, quadratic, exponential, or factorial, (b) calculate the number of instructions the algorithm would require to process all the pixels, (c) determine whether the algorithm is fast enough to meet the requirement of finishing a whole screen using 50 million instructions or less.

i. $T(n) = 1000 \lg n$

ii. $T(n) = 10n + 10000$

iii. $T(n) = n^2/2$

iv. $T(n) = 1.001^{n/10}$ (You may express your answer in scientific notation to two significant digits; also, consider using Google as your calculator if your handheld one balks.)
v. Suppose the company additionally wants to perform a smoothing operation on the entire screen’s display, where each pixel is given a value that is the average of its original value and those of its neighbors above, below, to the left, and to the right. Determine a function that gives the number of instructions this whole process will require. Then, as in the previous parts, (a) determine what type of function this running time is, (b) estimate how many instructions this will require, (c) declare this feasible or not. (Assume the pixel values are just single numbers indicating overall brightness. Addition or division can only be done two numbers at a time and takes a single instruction; assume arithmetic instructions account for all the instructions. In your estimation, you can ignore the fact that pixels at the edge have fewer neighbors.)