Written Homework 03

Assigned: Wed 05 Oct 2016
Due: Wed 19 Oct 2016

Instructions:

- The assignment has to be uploaded to blackboard by the due date. NO assignment will be accepted after 11:59pm on that day.

- We expect that you will study with friends and often work out problem solutions together, but you must write up your own solutions, in your own words. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.

- We require that all homework submissions be neat, organized, and typeset. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them by hand.

- To get full credit, show INTERMEDIATE steps leading to your answers, throughout.

Problem 1 [18pts, 6pts each] Password Troubles

You are generating random numeric 7-digit passwords in order to send them to the members of your website together with initial credentials. Your website is very successful, and you know that you will have at least 5 million people interested in becoming a member.

i. Assuming that you do not want to use “0” in your passwords, will you be able to generate enough passwords for your members? Why or why not?

ii. Suppose that due to a bug digits 3 and 8 (together with 0 that you don’t want anyway) are prevented from appearing in the passwords. How many different passwords would you be able to generate then?

iii. Unfortunately, the programmer who was supposed to fix the previous bug has introduced a new one. For some reason, his program generates now all possible passwords which have a “5” on exactly 3 of the 7 places (0 is still excluded, but all other digits can appear). How many passwords does his program generate?

Problem 2 [12pts, 6pts each]: Sets

Let A, B, and C be sets. Show that the below identities hold (in your work you can assume as given the standard laws of the set algebra, e.g., distributive laws, De Morgan’s, etc.).

i. $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$
ii. \((B \setminus A) \cup C = (B \cup C) \setminus (A \setminus C)\)

**Problem 3** [15 pts (4,5,6)]: Sets

i. Let \(A = \{y \mid y = 2x^2 + 11 \text{ for some } x \in \mathbb{N} \text{ which satisfies } x \leq 10\}\). List all elements of \(A\) (note: 0 is an element of \(\mathbb{N}\) for the purpose of this question).

ii. Let \(U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\) be the universe, \(A = \{1, 2, 4, 6, 8\}\), \(B = \{1, 2, 3, 7, 8, 9\}\), and \(C = \{2, 5, 8\}\) be three subsets of \(U\). What are the elements of the set \(\overline{A} \cup \overline{B} \cup \overline{C}\)?

iii. Let \(A = \{1, \{1\}, \{1, 1\}\}\). What are the elements of \(\mathcal{P}(A)\)? (\(\mathcal{P}(A)\) is the power set of \(A\))

**Problem 4** [30 pts;, (2,4,4,4,4,4,4,4,4)]: Divisibility

Consider the set \(S\) of natural numbers < 1000: \(S = \{0, 1, \ldots, 999\}\).

i. What is the cardinality of this set?

ii. How many of these integers are equal to \(1\)(mod 5)?

iii. How many of these integers are equal to \(1\)(mod 7)?

iv. How many of these integers are equal to \(1\)(mod 5) AND \(1\)(mod 7) simultaneously?

v. How many of these integers are equal to \(1\)(mod 6) and \(1\)(mod 15) simultaneously?

vi. What is the least number of distinct integers that need to be chosen from \(S\) so that at least one of them must equal \(1\)(mod 5)?

vii. What is the least number of distinct integers that need to be chosen from \(S\) so that at least one of them must equal \(1\)(mod 7)?

viii. What is the least number of distinct integers that need to be chosen from \(S\) so that at least one of them must equal \(1\)(mod 5) OR \(1\)(mod 7)

**Problem 5** [12 pts, (4,4,4)]: Power Company and Storms

Suppose that a power company maintains 75 transformers each in different county within a state. During contingency planning, the company considers inclement weather scenarios when some or all of the transformers fail. The company response may differ depending on exactly which transformers fail. Give the answer to the following questions and show your work.

i. How many possible failure scenarios are there (assume that none of the transformers’ failing is one such scenario)?

ii. If more than half of the transformers fail, the company will lose its license. How many scenarios are there where more than half of the transformers fail?
iii. In the middle of one particularly severe storm, 25 of the transformers have already failed and during an emergency meeting the company considers the likelihood of more than half of the transformers failing and their loosing the license. How many possible failure scenarios from that point onward are there which result in the company’s loosing their license? Is that more or less than a half of all the possibilities of the storm’s continuation? (Note: you can leave the number of failure scenarios unsimplified.)

Problem 6 [13 pts, (4,4,5)]: A TV Station’s Audience

Suppose that a polling company conducts a survey of the audience of a TV station. The company has surveyed a sample of 1000 of the station’s subscribers which consists of 450 men, 500 people 35 years of age or younger, and where 400 people of the sample have a college degree. They have also found that 150 of the men in the sample have a college degree, 250 women are over 35 years old, and that among the college graduates 250 were over 35. Finally, they were surprised at the relatively large number of uneducated men over 35 who subscribed to the station’s programming: 150.

i. An advertiser targets females or people who have a college degree. How many people like that are in the sample?

ii. Another advertiser targets men or people who do not have a college degree. How many people like that are there?

iii. The most lucrative demographics are the women who are at most 35 years of age, and who have a college degree. How many of such people are in the sample?