DIGITAL IMAGE PROCESSING (COM-3371)
Week 7 – February 25, 2002

Topics:

• Invited lecture: Dr. Simon Warfield (http://www.spl.harvard.edu/~warfield) will talk on "Exploiting Atlases in Medical Image Segmentation."

• Image representation and description

- Representation schemes
  - Chain codes
  - Polygonal approximations
  - Signatures
  - Boundary segments
  - The skeleton of a region
  - Line segmented encoding

- Boundary descriptors
  - Basic descriptors
  - Fourier descriptors

- Regional descriptors
  - Basic descriptors
  - Topological descriptors
  - Texture

• Morphology

• Term project - questions, problems, discussion

Readings:
• Chapter 11 (11.1 –11.3) and Chapter 9 (9.1-9.3. 9.5.1, 9.5.2) of text

IMAGE REPRESENTATION AND DESCRIPTION

INTRODUCTION

Region representation:

- based on external characteristics (its boundary)
- based on internal characteristics (pixels comprising the region)

Region description:

- boundary descriptors, such as boundary length, diameter, curvature, etc.
- regional descriptors, such as area, perimeter, compactness, mean value, etc.

Generally, an external representation is chosen when a primary focus is on ***shape*** *characteristics. An internal representation is selected when a primary focus is on* ***reflectivity properties, such as color or texture.***

REPRESENTATION SCHEMES

The segmentation techniques provide results in form of pixels along the boundary or pixels contained in a region:

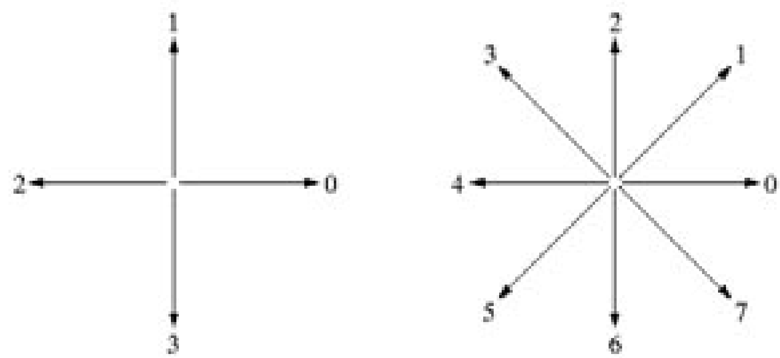
It is a standard practice to use boundary representation schemes that **compact** the data into representations that are more useful in the computation of descriptors. One of the schemes are chain codes.

Chain codes

- represent a boundary by a connected sequence of straight-line segments of specified length and direction
- the direction of each segment is coded by using a numbering scheme:

a b

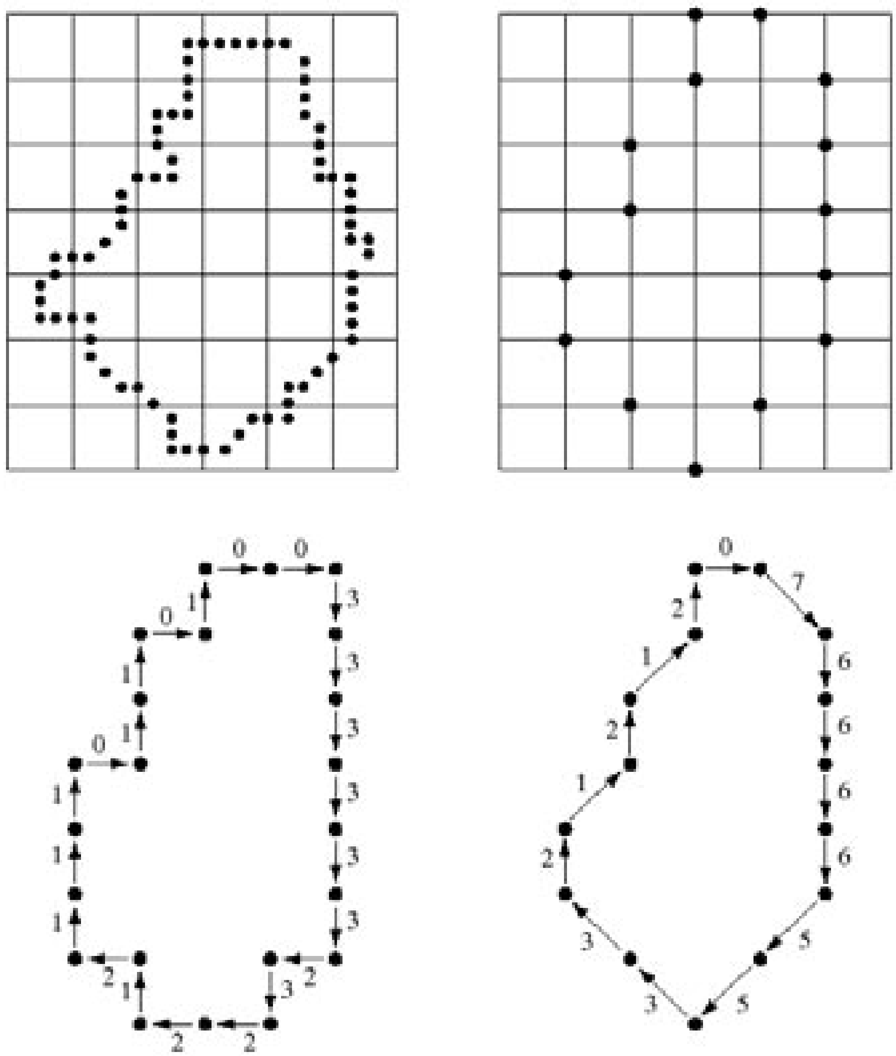
FIGURE 11.1 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.



- a typical chain code is generated by following a boundary in a clockwise direction and assigning a direction to the segments connecting every pair of pixels

  - this method results in quite long chain codes
  - any small disturbances along the boundary may cause changes in code that may not be related to the shape of the boundary

- another technique is to select a larger grid spacing, assign boundary points to each node of the large grid (depending on the proximity of the original boundary to that node), and represent the resampled boundary by a 4- 8-code:



a b
c d

FIGURE 11.2
(a) Digital boundary with resampling grid superimposed.
(b) Result of resampling.
(c) 4-directional chain code.
(d) 8-directional chain code.

the boundary representation in figure (c) is: 003333323221211101101
the boundary representation in figure (d) is: 076666553321212

The accuracy of the resulting code depends on the spacing of the sampling grid.

Polygonal approximations

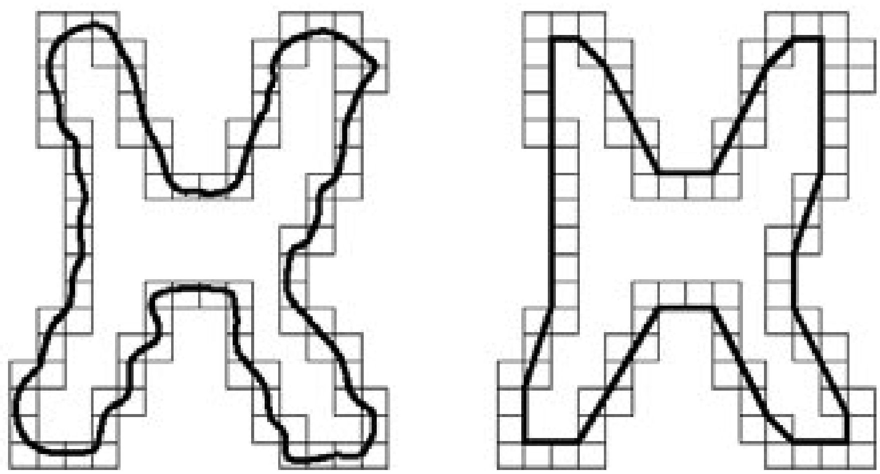
• a digital boundary can be approximated by a polygon

• **minimum perimeter** polygons

- enclose a boundary by a set of concatenated cells and produce a minimum perimeter that fits the cell strip:

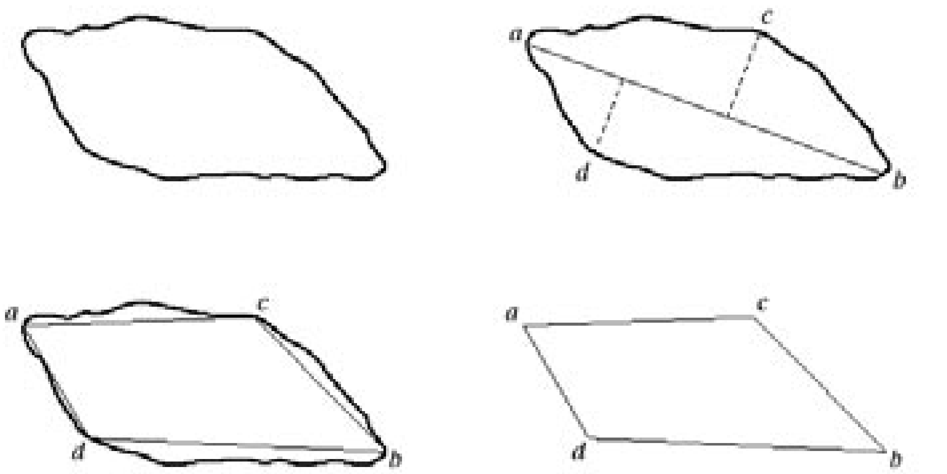
a b

FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.



- **merging** techniques (merge points along a boundary until the least square error line fit of the points merged exceeds a preset threshold; repeat the procedure for the new points along the boundary; at the end of the procedure the intersections of adjacent line segments form the vertices of the polygon)

- **splitting** techniques (subdivide a segment successively into two parts until a given criterion is satisfied -- for example, a requirement might be that the maximum perpendicular distance from a boundary segment to the line joining two end points does not exceed a preset threshold)



a b
c d

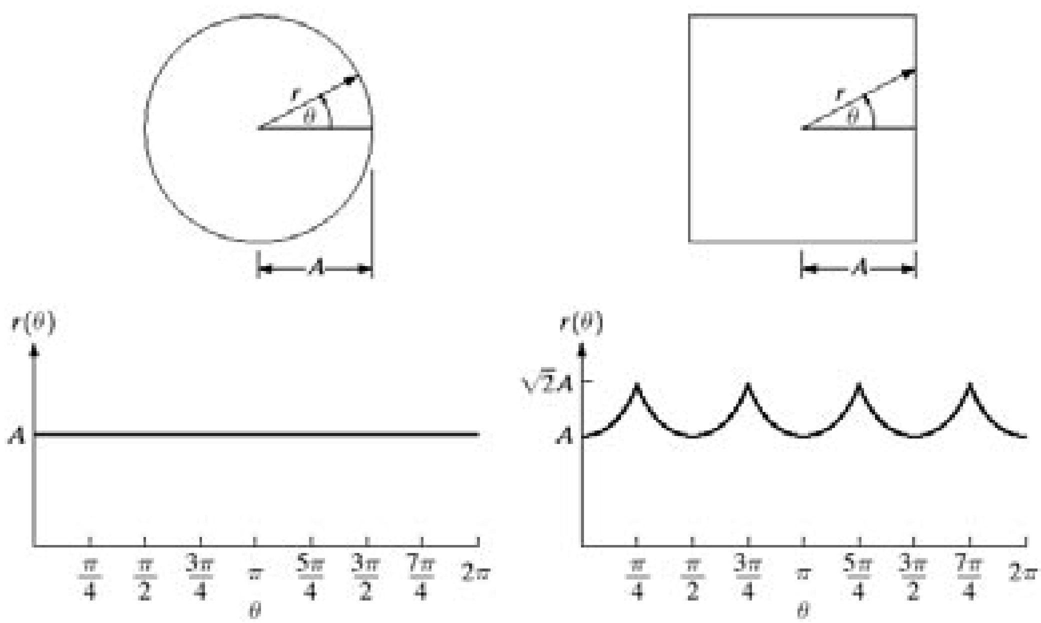
FIGURE 11.4
(a) Original boundary.
(b) Boundary divided into segments based on extreme points. (c) Joining of vertices.
(d) Resulting polygon.

Signatures

- a simple functional representation that can be used to describe and reconstruct the boundary with appropriate accuracy
- the simplest signature is a **plot of the distance from the centroid to the boundary as a function of angle**:

a b

FIGURE 11.5
Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \le \theta \le \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \le \pi/2$.



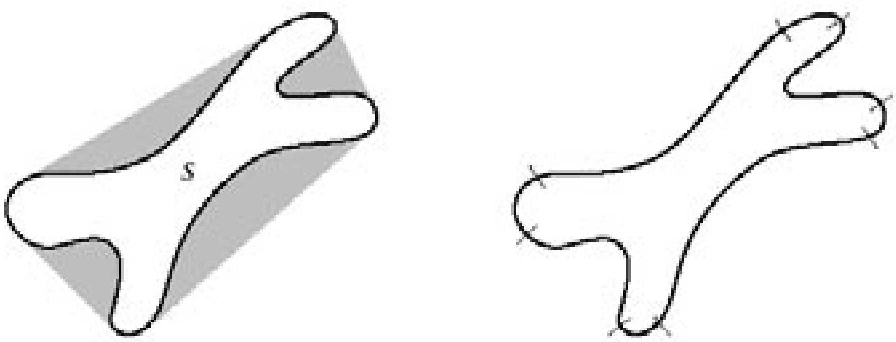
• signature is an translation-invariant representation which allows an object to be compared with a standard prototype by cyclically shifting the signature of one with respect to the other in steps, while checking for the best match

Boundary segments

• **convex hull** technique

- convex hull H of the arbitrary set S is the smallest convex set containing S; the set H-S is called convex deficiency D of the set S

- the region boundary can be partitioned by following the contour of S and marking the points at which a transition is made into or out of a component of the convex deficiency



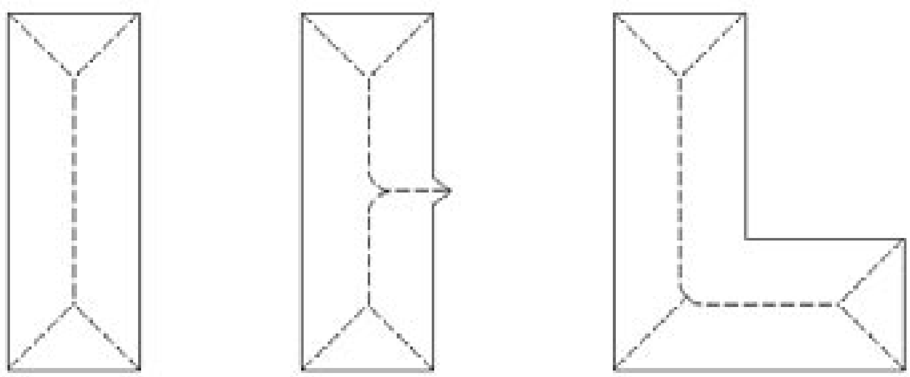
a b

FIGURE 11.6 (a) A region, S, and its convex deficiency (shaded). (b) Partitioned boundary.

- in practice, the boundary is usually smoothed prior to partitioning (for example, by using a polygonal approximation)

The skeleton of a region

- **medial axis transformation** (MAT) generates a "skeleton" of a region
- MAT algorithm:
  (1) for each point in the region we find its closest point in boundary,
  (2) if a point has more than one such a neighbor --> a point belongs to the medial axis (skeleton) of the region



a b c
FIGURE 11.7 Medial axes (dashed) of three simple regions.

- the results of MAT operation depend on the **distance measure**:

pixel	coordinates
p	(x,y)
q	(s,t)

Euclidean distance between p and q is defined as:
$D_e(p,q)=[(x-s)^2 + (y-t)^2]^{1/2}$

D4 distance:
$D_4(p,q)=|x-s|+ |y-t|$
For example, the pixels with D4 distance ≤ 2 from pixel (x,y) form the following contours of constant distance:

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

D_8 distance:

$D_8(p,q)=\max(|x-s|, |y-t|)$

For example, the pixels with D_8 distance ≤ 2 from pixel (x,y) form the following contours of constant distance:

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

• MAT operation is computationally expensive

BOUNDARY DESCRIPTORS

Length - number of pixels along the contour

X_{max}, X_{min}, Y_{max}, Y_{min} - maximum and minimum coordinates of the contour

$X_{centroid}$, $Y_{centroid}$ - X and Y coordinates of the center of the region

Boundary diameter - maximum distance between two points on the boundary

Curvature - rate of change of slope (example: the difference between the slopes of adjacent boundary segments)

Shape order (chain count) - number of elements in the chain

Shape number - first difference of smallest magnitude

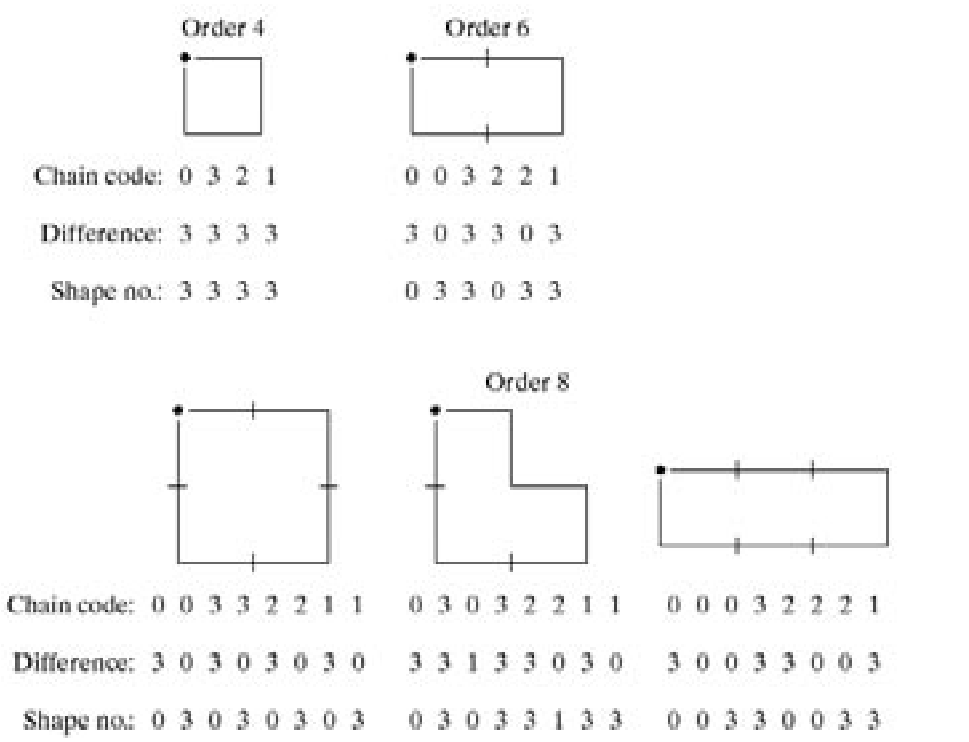
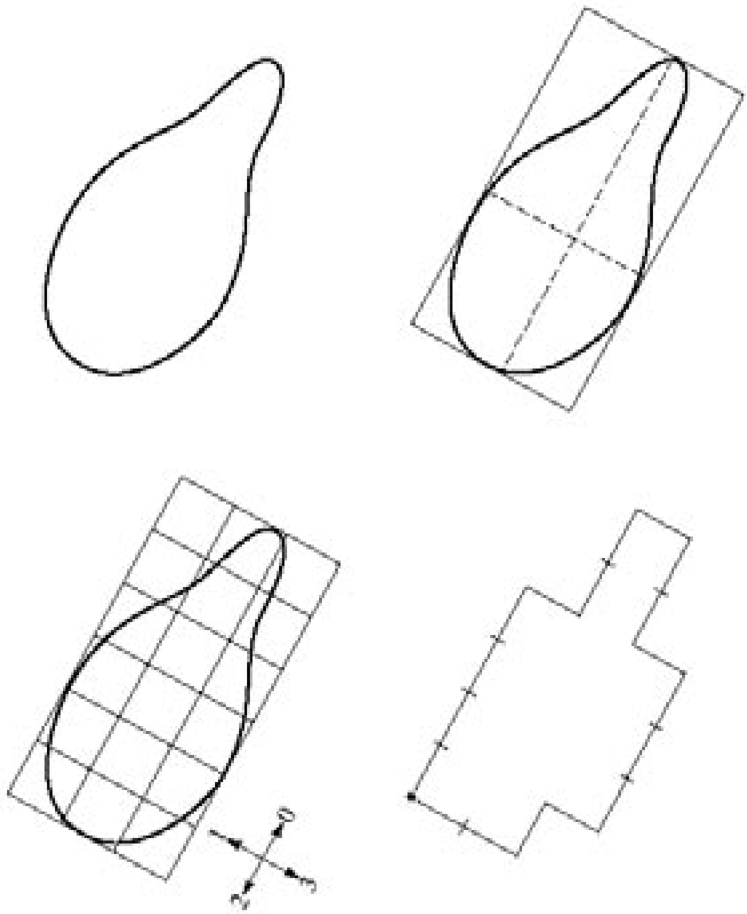


FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.



a b
c d

FIGURE 11.12 Steps in the generation of a shape number.

Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

Fourier descriptors

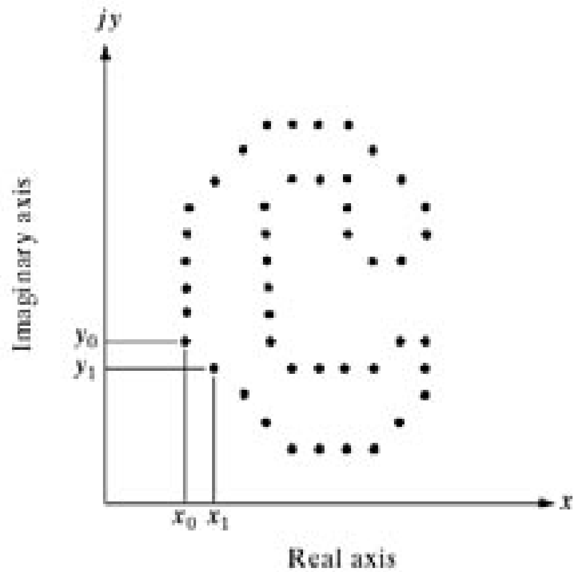
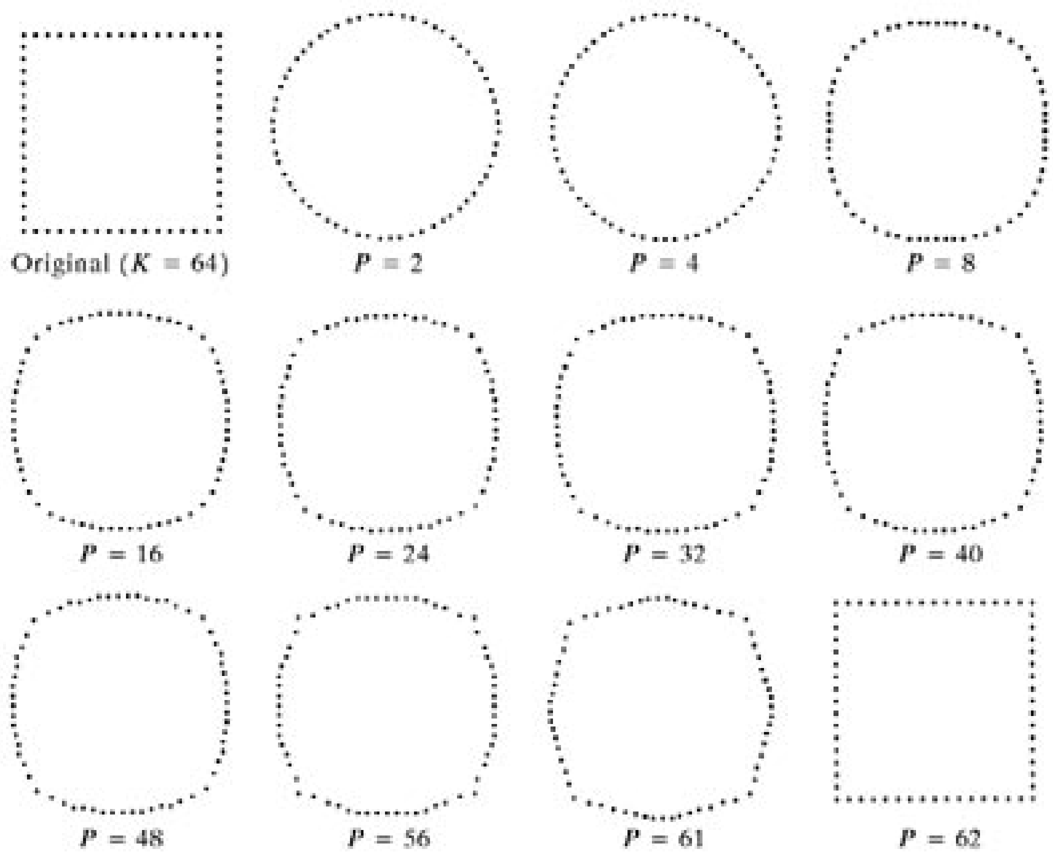


FIGURE 11.13 A digital boundary and its representation as a complex sequence. The points (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

- **the Fourier transform of a boundary representation** (chain code, signature, complex boundary function) is an alternative representation of the region's shape

- in many cases one can **lowpass filter** the boundary function spectrum without destroying the characteristic shape of the region --> this means that only the amplitudes and phases of the low frequency components in the spectrum (i.e. the low-order Fourier coefficients) are required to characterize the basic shape of the object and they can be used as shape descriptors

FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.



REGIONAL DESCRIPTORS

Perimeter - length of region's boundary

Area - number of pixels contained within its boundary

Shape factor - Perimeter^2/Area

Range - the difference between the largest and the smallest pixel values in a region

Median - median of pixels' intensity values within the region

Mean - average of pixels' intensity values within the region

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \overline{x}$$

Variance - the second moment about the mean

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \overline{x})^2}{N-1}$$

Standard deviation - square root of a variance

$$SD = \sqrt{\sigma^2}$$

Coef. of variation - a ratio of standard deviation to the mean

$$C = \frac{\sigma}{\mu}$$

Skewness - the third moment about the mean; an indicator of asymmetry in a pixel distribution

$$sk = \frac{\sum_{i=1}^{N} (x_i - \overline{x})^3}{(N-1)\sigma^3}$$

Kurtosis - the fourth moment about the mean; relates to the degree of peakedness or flatness of a distribution (for the normal distribution kurtosis =3)

$$kurt = \frac{\sum_{i=1}^{N} (x_i - \overline{x})^4}{(N-1)\sigma^4}$$

Topological descriptors

• topology - the study of properties of a figure that are unaffected by any deformation except tearing or folding

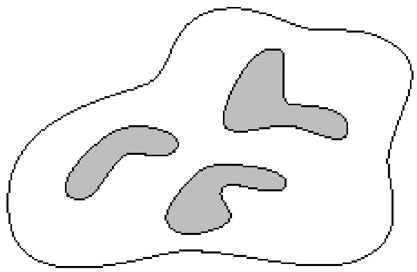


FIGURE 11.18 A region with three connected components.

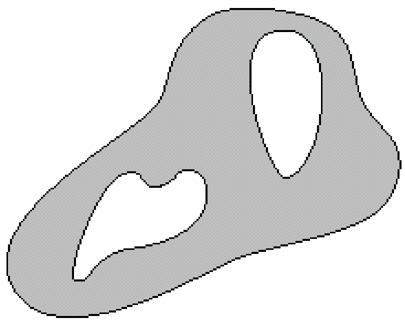


FIGURE 11.17 A region with two holes.

• topological features give a **global** description of a region

examples:

number of holes

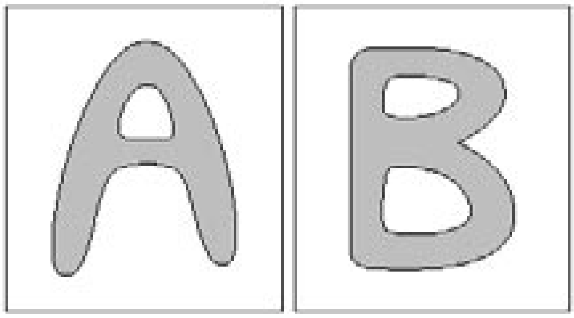
number of connected components

number of object edges

number of faces

number of vertices

Euler number (the difference between the number of connected components and number of holes)

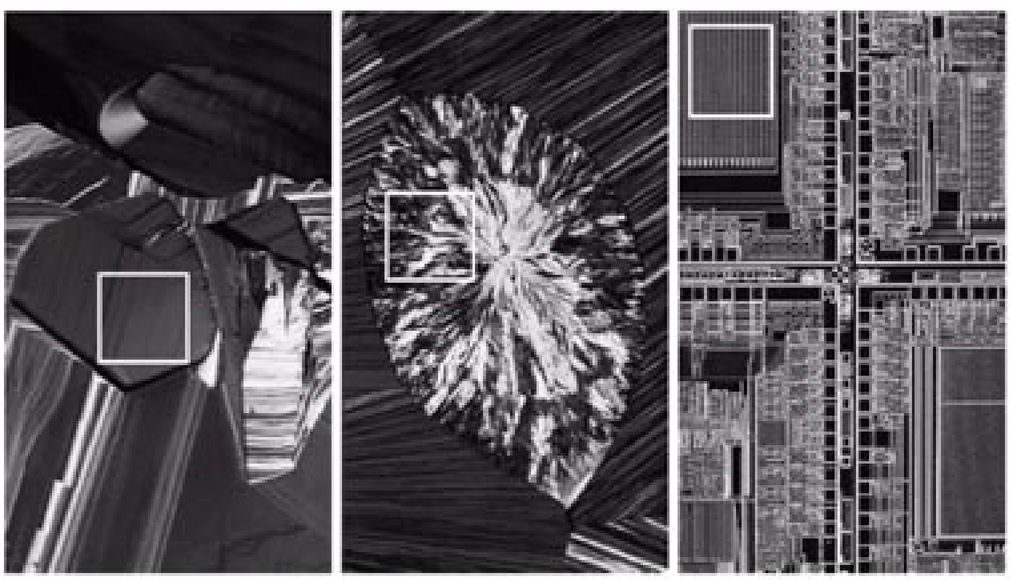


a b

FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

Texture

- texture, as observed in wood grain, stone, cloth, grass, etc., is an important region description
- no formal definition; intuitively this descriptor provides measures of properties, such as smoothness, coarseness, regularity, etc.



a b c

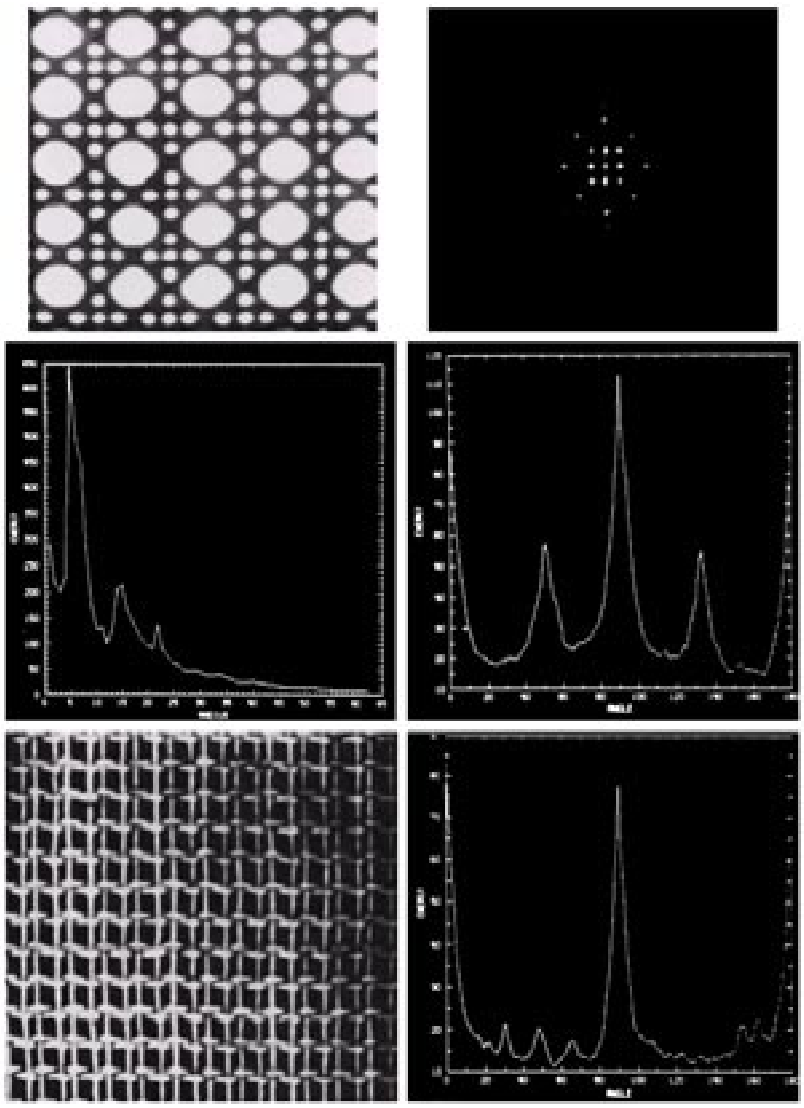
FIGURE 11.22 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

• three principal approaches used to describe the texture are:

statistical - smoothness, coarseness, graininess (using moments, entropy, etc.)

structural - arrangement of image primitives, such as regularity of parallel lines

spectral - based on properties of Fourier spectrum (periodic patterns in an image)



a b
c d
e f

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

MORPHOLOGY

- Morphological operators - tools for extracting image components that are useful in the representation and description of region shape (examples: erosion, dilation, etc.)

- The language of mathematical morphology is **set theory**

- **Erosion** - removes pixels from the periphery of a region (it also removes single pixels)

- **Dilation** - adds a layer of pixels around a periphery of a region (it also fill small holes within regions)

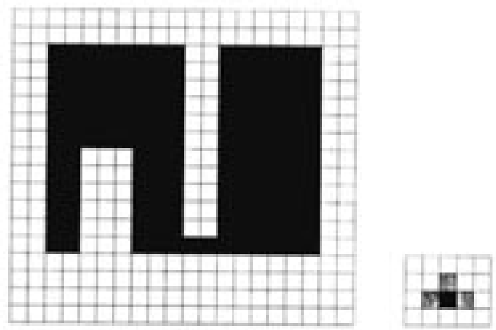


Figure 2.25: The original test image A (left) and structuring element B (right). Note that the origin of the structuring element is darker than the other pixels in B.

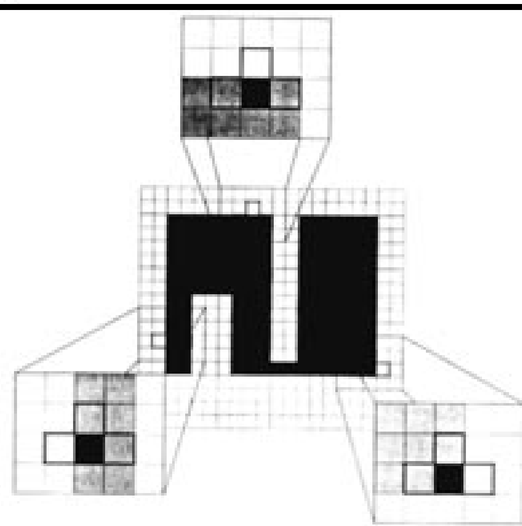
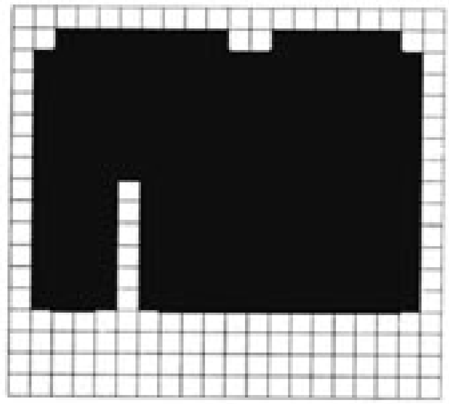


Figure 2.26: Translations of the structuring element B to 1 pixels in A where the entire structuring element is *not* contained within A. During a dilation operation, every pixel in the structuring element will be present in the final image. During an erosion operation, the pixel at the origin of the structuring element will be deleted.

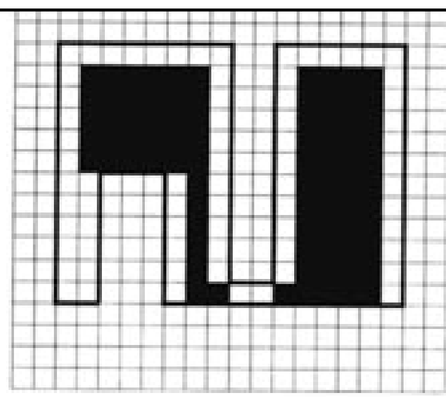
If A_{b_i} are translations of the binary image A by 1 pixels of the binary image B, then the union of the translations of A by the 1 pixels of B is called the dilation of A by B.



$$A \oplus B = \bigcup_{b_i \in B} A_{b_i}$$

Figure 2.27: The dilation of A by B. The boundary of the original figure A is shown as a bold line.

The erosion of a binary image A by a binary image B is 1 at a pixel p if and only if every 1 pixel in the translation of B to p is also in A



$$A \ominus B = \{p | B_p \subseteq A\}$$

Figure 2.28: The erosion of A by B. The boundary of the original figure A is shown as a bold line.

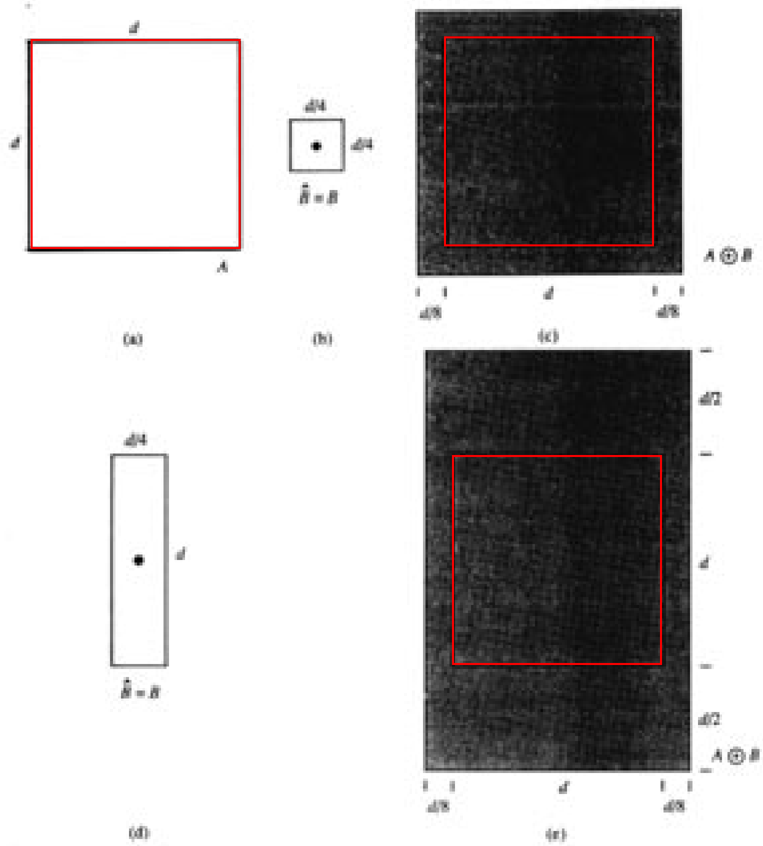


Figure 8.26 *(a) Original set A; (b) square structuring element and its reflection; (c) dilation of A by B, shown shaded; (d) elongated structuring element; (e) dilation of A using this element.*

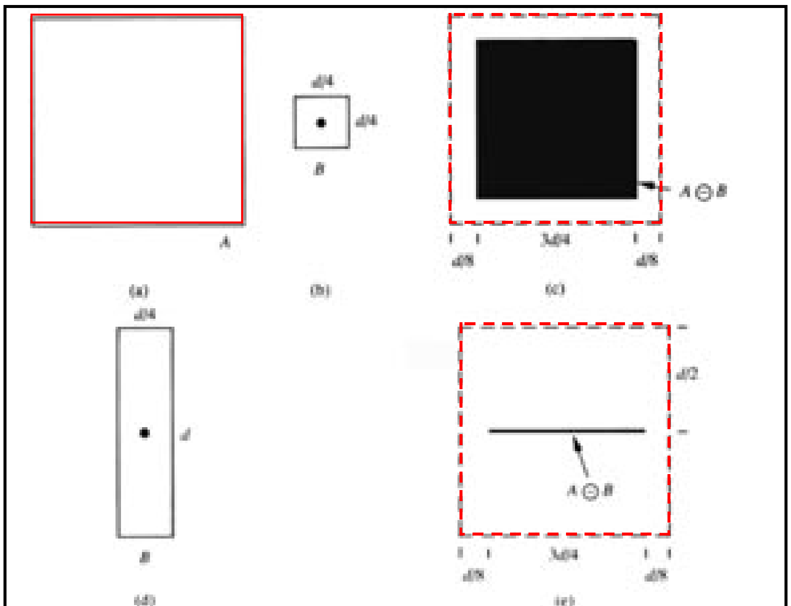


Figure 8.27 *(a) Original set A; (b) structuring element B; (c) erosion of A by B, shown shaded; (d) elongated structuring element; (e) erosion of A by this element.*

- Erosion and dilation are often used in **filtering** the images - if the nature of noise is known, then a suitable structuring element can be used and a sequence of erosion and dilation operations can be applied for removing the noise

- **Opening** - a combination of an erosion followed by a dilation (opening up the spaces between touching regions, removing pixels in regions which are too small to contain the structuring element)

- **Closing** - a combination of an dilation followed by an erosion (fusing narrow brakes, eliminating small holes, filling gaps smaller than the structuring element)

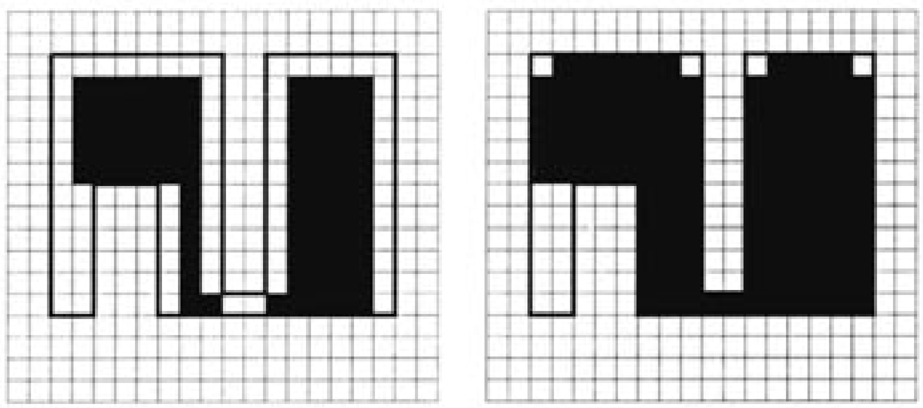


Figure 2.32: Opening operation. *Left:* Initial erosion. *Right:* Succeeding dilation. The boundary of the original figure A is shown as a bold line.

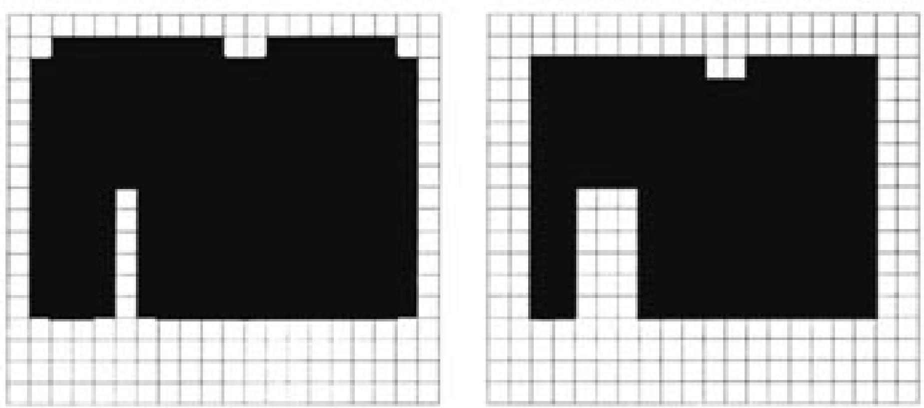
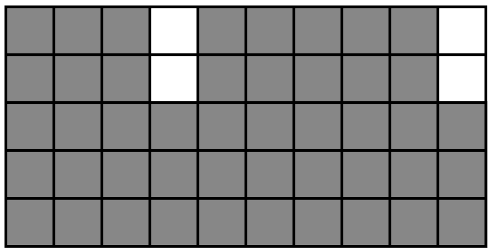
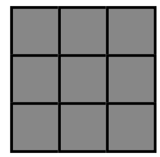


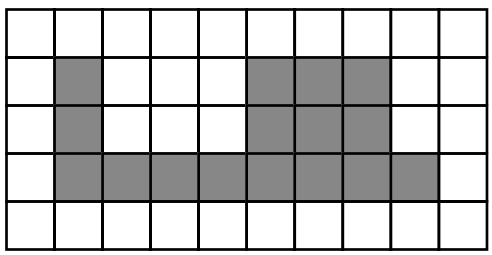
Figure 2.33: Closing operation. *Left:* Initial dilation. *Right:* Succeeding erosion. The boundary of the original figure A is shown as a bold line.

- The structuring element (probe) does not have to be compact or regular - it can be any pattern of pixels
- Morphological operations can be used for **boundary extraction**:

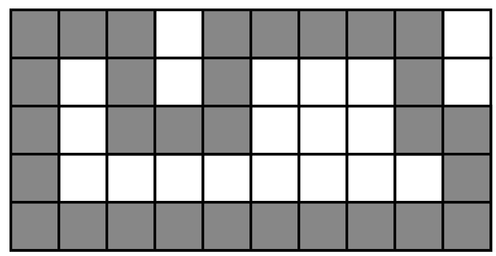


Set A





A eroded by B



Boundary extracted by taking the set difference between A and its erosion

- Morphological operations can also be used for **region filling** and for **extraction of connected components**

- Morphological operations can be used for **optical character recognition**:
    a. Create a model for each character:
       - Extract the character to be recognized
       - Use expanding or closing to fill holes and cavities
       - Shrink the character image to remove unwanted regions and to reduce the size so that it will fit inside an instance of the character
    b. Preprocess the character image (fill the holes, remove unwanted pixels)
    c. Use the character model as a structuring element and perform erosion
    d. Compute the connected components
    e. Apply the size filter to discard regions that are too small
    f. Compute the position of each region that passes through the size filter --> this provides the position of each recognized instance of the character model in the image