DIGITAL IMAGE PROCESSING (COM-3371)
Week 6- February 18, 2002

Topics:

• Image restoration (continuation)
  • Periodic noise reduction by frequency domain filtering
    • Bandreject filters
    • Bandpass filters
    • Notch filters
• Image segmentation
  • Introduction
  • Thresholding
    • Multilevel thresholding
    • Smoothing and thresholding
    • Optimal thresholding
    • Thresholding based on local properties
    • Dynamic (adaptive) thresholding
• The watershed algorithm
• Region growing
• Edge-based segmentation

• Term projects - discussions

Homework:
Homework 3 is due February 25, 2002

Readings:
• Chapter 5 (5.4.1-5.4.3) and Chapter 10 (10.1, 10.3.1-10.3.4, 10.3.7, 10.4.1-10.4.2, 10.5.1) of text.
• Reprints given in class (on segmentation)
IMAGE RESTORATION (continuation)

Periodic noise reduction by frequency domain filtering

• During the lecture on Fourier transform, we discussed lowpass and highpass filters. The simplest lowpass filter (ideal) was defined as (Eq. 4.3-2 in G&W):

\[
H(u, v) = \begin{cases} 
1 & \text{if } D(u, v) \leq D_0 \\
0 & \text{if } D(u, v) > D_0 
\end{cases}
\]

(1)

where \( D_0 \) is a specified nonnegative quantity, \( D(u, v) \) is the distance from point \((u,v)\) to the origin of the frequency rectangle.

This filter cuts off all high frequency components of the Fourier transform that are at a distance greater than distance \( D_0 \) from the origin of the centered transform.

• If the image size is \( M \times N \), the transform is also of this size.

• The center of the frequency rectangle is at \((u,v) = (M/2, N/2)\) due to the fact that the transform has been centered.

• The distance from any point \((u,v)\) to the center (origin) of the Fourier transform is given by:

\[
D(u, v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}
\]

(2)

(see Fig. 4.10, p. 168 in G&W)
• Similarly, high-pass filter was defined (Eq. 4.4-2):

\[ H(u, v) = \begin{cases} 
0 & \text{if } D(u, v) \leq D_0 \\
1 & \text{if } D(u, v) > D_0 
\end{cases} \]  

(3)

(see Fig. 4.22, p. 181 in G&W)

**BANDREJECT FILTERS**

- **Band reject filters** --- remove or attenuate a band of frequencies about the origin of the Fourier transform
- **Ideal bandreject filter:**

\[ H(u, v) = \begin{cases} 
1 & \text{if } D(u, v) < D_0 - \frac{W}{2} \\
0 & \text{if } D_0 - \frac{W}{2} \leq D(u, v) \leq D_0 + \frac{W}{2} \\
1 & \text{if } D(u, v) > D_0 + \frac{W}{2} 
\end{cases} \]  

(4)
A Butterworth bandreject filter is defined:

\[ H(u, v) = \frac{1}{1 + \left( \frac{D(u, v)W}{D^2(u, v) - D_0^2} \right)^{2n}} \]  

where \( D(u,v) \) is the distance from the origin of the centered frequency rectangle, and \( D_0 \) is the cutoff frequency.
A Gaussian bandreject filter is defined:

\[
H(u, v) = 1 - e^{-\frac{1}{2} \left[ \frac{D^2(u, v) - D_0^2}{D(u, v)W} \right]^2}
\]

Example of bandreject filter application:

See Fig. 5.16, p. 245 in G&W

BANDPASS FILTERS

A bandpass filter performs the opposite operation of a bandreject filter and can be defined as:

\[
H_{bp}(u, v) = 1 - H_{br}(u, v)
\]

where \(H_{bp}\) is the transfer function of the bandpass filter, and \(H_{br}\) is the transfer function of the bandreject filter.

The bandpass filter is useful in isolating the effect on an image of selected frequency bands (see Fig. 5.17 in G&W).
NOTCH FILTERS

- A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency.
• Due to the symmetry of the Fourier transform, notch filter must appear in symmetric pairs about the origin in order to obtain meaningful results (exception: notch filter located in the origin)
• The number of pairs of notch filters can be arbitrary; the shape of the notch areas can also be arbitrary
• The transfer function of the ideal notch reject filter of radius $D_0$ with centers at $(u_0,v_0)$ and, by symmetry, at $(-u_0,-v_0)$ is:

\[
H(u, v) = \begin{cases} 
0 & \text{if } D_1(u, v) \leq D_0 \text{ or } D_2(u, v) \leq D_0 \\
1 & \text{otherwise} 
\end{cases}
\]

where

\[
D_1(u, v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}
\]

and

\[
D_2(u, v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}
\]

The assumption is that the center of the frequency rectangle has been shifted to the point $(M/2,N/2)$; therefore, the values of $(u_0,v_0)$ are with respect to the shifted center.
• A Butterworth notch reject filter:

\[ H(u, v) = \frac{1}{1 + \left[ \frac{D_0}{D_1(u, v) - D_2(u, v)} \right]^n} \]

where \( D_1 \) and \( D_2 \) are given by Equations 9 and 10

(11)

• A Gaussian notch reject filter:

\[ H(u, v) = 1 - e^{-\frac{1}{2} \left[ \frac{D_1(u, v)D_2(u, v)}{D_0^2} \right]} \]

(12)

• Notch filters that pass frequencies contained in the notch areas (notch-pass filters) perform exactly the opposite function as the notch reject filters:

\[ H_{np}(u, v) = 1 - H_{nr}(u, v) \]

(13)

• Example
  (see Fig. 5.19, p. 249 in G&W)